Quantum Gravity from Dynamical Metric Fluctuations

来自动力学度规涨落的量子引力

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Abstract

摘要

In this contribution, we discuss the asymptotic safety scenario for quantum gravity with a functional renormalization group approach that disentangles dynamical metric fluctuations from the background metric. We detail the derivation of flow equations on Euclidean and Lorentzian signatures and discuss the diffeomorphism symmetry constraints on the flow. Concerning the results, we focus on a comprehensive review of momentum-dependent correlation functions at vanishing cutoff scale, the asymptotically safe Standard Model, and spectral properties of asymptotically safe gravity from direct computation in space-times with Lorentzian signatures.

在本文中，我们利用功能重正化群方法讨论量子引力的渐近安全情景，该方法可将动力学度量涨落与背景度量分离开。我们详细推导了欧几里得号差和洛伦兹号差下的流方程，并讨论了微分同胚对称性对流的约束。关于研究结果，我们聚焦于全面综述以下内容: 截止尺度为零时依赖动量的关联函数、渐近安全标准模型，以及洛伦兹号差时空直接计算得到的渐近安全引力谱性质。

Keywords

关键词

Asymptotically safe quantum gravity - Functional renormalization group . Lorentzian quantum gravity - UV completion of the Standard Model

渐近安全量子引力-泛函重整化群。洛伦兹量子引力-标准模型的紫外完备性

Introduction

引言

The unification of general relativity with quantum field theory remains one of the most fundamental problems of modern physics. Despite decades of research, the question of whether the metric field can be used as a fundamental degree of freedom in a quantum field theory has not been conclusively answered yet. While perturbative quantizations of metric quantum gravity run into problems of predictivity or unitarity, it has been pointed out by Weinberg [1] that gravity might possess a nontrivial ultraviolet (UV) fixed point of the renormalization group (RG) flow. This UV fixed point would make the theory non-perturbatively renormalizable.

广义相对论与量子场论的统一仍是现代物理学最基础的核心问题之一。历经数十年研究，度规场能否作为量子场论中的基本自由度这一问题仍未有定论。虽然度规量子引力的微扰量子化面临可预言性或幺正性问题，但温伯格 [1] 早已指出，引力可能在重整化群 (RG) 流中存在非平凡的紫外 (UV) 不动点，该紫外不动点可让该理论具备非微扰可重整性。

The method of choice for respective investigations is the functional renormalization group (fRG) in its form for the effective action [2]. The fRG approach to quantum gravity has been initiated by the seminal paper [3], where the UV fixed point has been studied in the Einstein-Hilbert truncation. In this approximation, one retains only two couplings, the Newton coupling G_{N} and the cosmological constant Λ . Already this basic truncation exhibits a UV fixed point in four dimensions; see [3,4]. This exciting finding has triggered a plethora of works for asymptotically safe gravity with and without matter, and we refer the reader to the textbooks [5, 6] and reviews [7-18]. For very recent accounts of the challenges for asymptotically safe gravity, see [19, 20]. For generic reviews on the fRG, we refer to [21-30].

开展相关研究的首选方法是有效作用量形式下的泛函重整化群 (fRG)[2]。量子引力的 fRG 方法由开创性文献 [3] 开创，该文献在爱因斯坦-希尔伯特截断下研究了紫外不动点。在该近似中，仅保留两个耦合常数: 牛顿耦合 G_{N} 和宇宙学常数 Λ 。即使是这个基础截断，在四维中也存在紫外不动点，参见文献 [3,4]。这一令人振奋的发现催生了大量关于含物质和不含物质渐近安全引力的研究，相关内容读者可参阅专著 [5,6] 与综述文献 [7-18]。关于渐近安全引力所面临挑战的最新论述可参见 [19,20]，关于 fRG 的通用综述参见 [21-30]。

In this contribution, we are discussing the fluctuation approach to metric quantum gravity. The key property of this approach is that the metric background and the fluctuations about it are strictly disentangled. The RG running is then set up in terms of correlation functions of the fluctuation and background field. The approach has been reviewed in [31] to which we refer for extended details and more background information. Here we concentrate on new developments in the approach, which includes the first results on Lorentzian backgrounds within the spectral fRG, full 1PI quantum correlation functions, and the UV and infrared (IR) physics of asymptotically safe matter-gravity systems including the full Standard Model (SM).

本文将讨论度规量子引力的涨落方法。该方法的核心特点是将背景度规与度规围绕背景的涨落严格解耦，重整化群跑动由涨落场和背景场的关联函数来描述。该方法的详细内容与更多背景信息已在综述 [31] 中介绍，本文将聚焦该方向的最新进展，包括谱 fRG 框架下洛伦兹背景的首个结果、完整的 1PI 量子关联函数，以及包含完整标准模型 (SM) 的渐近安全物质-引力系统的紫外和红外 (IR) 物理。

Quantum Field Theory Approach to Quantum Gravity

量子引力的量子场论方法

The quantum field theory approach to metric quantum gravity is based on the path integral over the metric fluctuations for a given classical gravity action. This reads schematically

度规量子引力的量子场论方法，基于对给定经典引力作用量的度规涨落进行路径积分。其概要形式如下

$$\int \mathcal{D}\hat{g}_{\mu\nu} e^{\mathrm{i}S_{\text{grav}}[\hat{g}_{\mu\nu}]} \tag{1}$$

where the hat indicates fields that are integrated over, which is here only the metric field $\hat{g}_{\mu\nu}$. In general, the gravitational action in (1) also includes matter fields and an integration over these fields. A typical choice for the classical gravitational action is the Einstein-Hilbert action:

其中带帽符号表示被积场，此处仅为度规场 $\hat{g}_{\mu\nu}$ 。一般而言，(1) 中的引力作用量还包含物质场，也需要对这些场做积分。经典引力作用量的典型选择是爱因斯坦-希尔伯特作用量:

$$S_{\text{EH}}\left[g_{\mu\nu}\right] = \frac{1}{16\pi G_{\text{N}}} \int \mathrm{d}^4 x \sqrt{-g}\,(R - 2\Lambda)\,,\ g = \det g_{\mu\nu}, \tag{2}$$

where G_{N} is the classical Newton coupling, Λ is the cosmological constant, and R is the Ricci scalar with

其中 G_{N} 是经典牛顿耦合常数，Λ 是宇宙学常数，R 是里奇标量，由下式计算

$$R = R^{\mu}{}_{\mu},\ R_{\mu\upsilon} = R^{\lambda}{}_{\mu\lambda\upsilon}, \tag{3}$$

computed from the Riemann tensor

由黎曼张量得到

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}, \tag{4}$$

with the Levi-Civita connection:

其中联络为列维-奇维塔联络:

$$\Gamma^{\sigma}{}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right). \tag{5}$$

Quantum gravity based on the Einstein-Hilbert action (2) is perturbatively non-renormalizable; see [32-35], which is related to the negative mass dimension of the Newton constant in four space-time dimensions, $[G_\mathrm{N}] = -2$.

基于爱因斯坦-希尔伯特作用量 (2) 的量子引力在微扰下不可重整，参见文献 [32-35]；这与四维时空下牛顿常数 $[G_\mathrm{N}] = -2$ 的质量量纲为负有关。

One can construct theories of quantum gravity that are perturbatively renormal-izable when higher-order curvature invariants are included in the classical action. A prominent example is provided by the Stelle action [36,37]:

在经典作用量中引入高阶曲率不变量后，可以构造出微扰可重整的量子引力理论。一个典型的例子是施泰勒作用量 [36,37]:

$$S_{\text{Stelle}}\left[g_{\mu\nu}\right] = S_{\text{EH}} + \int \mathrm{d}^4 x \sqrt{-g} \left(aR^2 + bC^2_{\mu\upsilon\rho\sigma} + cE \right), \tag{6}$$

where $C_{\mu\upsilon\rho\sigma}$ is the Weyl tensor and E is the Gauß-Bonnet density, given by $E = \frac{1}{32\pi^2}\left(R^2 - 4R^{\mu\upsilon}R_{\mu\upsilon} + R^{\mu\upsilon\rho\sigma}R_{\mu\upsilon\rho\sigma}\right)$, which is a topological term in four dimensions. This classical action is perturbatively renormalizable due to the dimensionless couplings a, b , and c . However, it contains a massive ghost state that might spoil unitarity [36-38].

其中 $C_{\mu\upsilon\rho\sigma}$ 是外尔张量，E 是高斯-博内密度，由 $E = \frac{1}{32\pi^2}\left(R^2 - 4R^{\mu\upsilon}R_{\mu\upsilon} + R^{\mu\upsilon\rho\sigma}R_{\mu\upsilon\rho\sigma}\right)$ 给出，它在四维空间中是拓扑项。由于耦合常数 a, b 和 c 是无量纲的，该经典作用量具有微扰可重整性，但它包含一个大质量鬼态，可能会破坏幺正性 [36-38]。

The perturbative failure of making sense out of the gravitational path integral (1) does not imply that quantum gravity cannot be described by a fundamental metric field. Instead, it emphasizes the need for non-perturbative methods such as lattice and functional methods. The key idea is that the existence of an interacting UV fixed point makes the theory non-perturbatively renormalizable [1]. This UV fixed point exists in $2 + \varepsilon$ dimensions and might extend up to $d = 4$ dimensions [39-45]. The four-dimensional case was tested with lattice computations [46-53] and fRG [2, 54, 55] ; see [30] for a review on the fRG. The first indications for the UV fixed point in four dimensions were found with the fRG in [3,4].

引力路径积分 (1) 在微扰框架下的失效，并不意味着量子引力无法由基本度规场描述，反而说明我们需要非微扰方法，例如格点方法和泛函方法。其核心思想是，相互作用紫外不动点的存在使得该理论在非微扰意义下可重整 [1]。该紫外不动点存在于 $2 + \varepsilon$ 维中，且可以延拓到 $d = 4$ 维 [39-45]。四维的情况已经通过格点计算 [46-53] 和 fRG [2, 54, 55] 进行了检验；关于 fRG 的综述可参见 [30]。四维空间中存在紫外不动点的首个迹象由文献 [3,4] 通过 fRG 得到。

In the fRG approach pursued in the present work, the key object is the full quantum effective action $\Gamma\left[g_{\mu\nu}\right]$ with $g_{\mu\nu} = \langle \hat{g}_{\mu\nu} \rangle$, and the UV closure of the theory is obtained through a UV fixed point with finitely many relevant directions. The respective fixed-point effective action is a highly nontrivial functional of the (mean) metric, and the fixed-point analysis in such a framework only depends on the field content: the underlying theory is fixed by the chosen UV fixed point or rather its respective UV-relevant directions. It is this choice and the selected trajectory emanating from the fixed point that determines the theory and hence the underlying classical or rather IR-effective action. For example, one can find fixed points that relate to an IR action that only features the cosmological-constant term, which is a non-dynamical IR theory. Furthermore, one can find fixed points that contain the spin-two ghost of the Stelle gravity (6) and others that do not. We are interested in finding a dynamical IR effective theory without ghost states.

在本文所采用的 fRG 框架中，核心研究对象是带 $g_{\mu\nu} = \langle \hat{g}_{\mu\nu} \rangle$ 的全量子有效作用量 $\Gamma[g_{\mu\nu}]$，理论的紫外闭合通过具有有限多个相关方向的紫外不动点实现。对应的不动点有效作用量是 (平均) 度规的高度非平凡泛函，该框架下的不动点分析仅依赖场的内容: 基础理论由所选的紫外不动点，或者说由其对应的紫外相关方向确定。正是这一选择，以及从不动点出发的所选轨迹决定了整个理论，进而决定了基础的经典，即红外有效作用量。例如，我们可以找到对应仅含宇宙学常数项的红外作用量的不动点，该红外理论是非动力学的。此外，有些不动点包含施泰勒引力 (6) 的自旋二鬼态，另一些则不含。我们的目标是找到不存在鬼态的动力学红外有效理论。

The fRG approach as well as the metric path integral in (1) is based on the existence of the metric propagator. For the latter to exist, the diffeomorphism invariance of gravity has to be factored out, which is done within the Faddeev-Popov procedure. Then, the classical action in (1) is supplemented with a gauge-fixing and ghost term. The latter terms (as do all source terms) require the introduction of a background metric $\bar{g}_{\mu\nu}$. The expansion of the full metric $g_{\mu\nu}$ about this background introduces the fluctuation field $h_{\mu\nu}$ as the dynamical field in this expansion. Accordingly, it is the fluctuation field that is integrated over in the path integral. Throughout this contribution, we only consider the linear metric split:

fRG 方法与式 (1) 中的度规路径积分都基于度规传播子的存在。要使度规传播子存在，必须分解出引力的微分同胚不变性，这一步骤在法捷耶夫-波波夫 (Faddeev-Popov) 流程中完成。随后，式 (1) 中的经典作用量会补充规范固定项和鬼项。这些项 (与所有源项一样) 需要引入背景度规 $\bar{g}_{\mu\nu}$ 。全度规 $g_{\mu\nu}$ 围绕该背景的展开将涨落场 $h_{\mu\nu}$ 引入为该展开中的动力学场。相应地，路径积分中积分的对象正是涨落场。本文通篇仅考虑线性度规分裂:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{7}$$

This split is by no means unique, and various metric splits have been considered in the literature, for example, the exponential split [41, 56-71]. More elaborate choices are typically based on the geometrical properties of gravity as pioneered by Vilkovisky and DeWitt [72-75], which was also used in asymptotic safety with the fRG; see [24, 76-80]. For a more detailed discussion on the splits, we refer to [31].

这种分裂绝非唯一，文献中已经研究了多种度分裂，例如指数分裂 [41, 56-71]。更精巧的选择通常基于维尔科维斯基 (Vilkovisky) 和德维特 (DeWitt)[72-75] 开创的引力几何性质，这种选择也已被用于 fRG 框架下的渐近安全研究；参见 [24, 76-80]。关于度规分裂的更详细讨论，我们推荐读者参阅 [31]。

A typical choice for the gauge-fixing action is a linear gauge-fixing condition for the fluctuation field $h_{\mu\nu}$
:

规范固定作用量的一个典型选择是涨落场 $h_{\mu\upsilon}$ 的线性规范固定条件:

$$S_{\mathrm{gf}}[\bar{g}, h] = \frac{1}{2\alpha}\int \mathrm{d}^4x\sqrt{-\bar{g}}\bar{g}^{\mu\upsilon}F_\mu F_\upsilon. \tag{8}$$

A common gauge-fixing condition F_μ is given by

一个常用的规范固定条件 F_μ 由下式给出

$$F_\mu[\bar{g}, h] = \overline{\nabla}^\upsilon h_{\mu\upsilon} - \frac{1+\beta}{4}\overline{\nabla}_\mu h^\upsilon_\upsilon, \tag{9}$$

where $\overline{\nabla}$ is the covariant derivative with the connection (5) of the background metric $\bar{g}_{\mu\nu}$. The gauge-fixing (9) is introduced in the path integral with the Faddeev-Popov trick and the Jacobi determinant of the respective reparameterization. The Faddeev-Popov determinant can be rewritten in terms of a fermionic path integral with the ghost fields c_μ and $\bar{c}_\mu$. The ghost action related to (9) reads

其中 $\overline{\nabla}$ 是带背景度规 $\bar{g}_{\mu\nu}$ 联络 (5) 的协变导数。式 (9) 的规范固定通过法捷耶夫-波波夫技巧和相应重参数化的雅可比行列式引入路径积分。法捷耶夫-波波夫行列式可以改写为带鬼场 c_μ 和 $\bar{c}_\mu$ 的费米路径积分。与式 (9) 对应的鬼作用量为

$$S_{\mathrm{gh}}[\bar{g}, h, c, \bar{c}] = \int \mathrm{d}^4x\sqrt{-\bar{g}}\bar{c}^\mu M_{\mu\nu}c^\upsilon, \tag{10}$$

with the Faddeev-Popov operator

其中法捷耶夫-波波夫算子为

$$M_{\mu\upsilon} = \overline{\nabla}^\rho\left(g_{\mu\upsilon}\nabla_\rho + g_{\rho\upsilon}\nabla_\mu\right) - \frac{1+\beta}{2}\bar{g}^{\sigma\rho}\overline{\nabla}_\mu g_{\upsilon\sigma}\nabla_\rho, \tag{11}$$

which contains both the background covariant derivative $\overline{\nabla}$ and the covariant derivative ∇ with the connection (5) of the full metric $g_{\mu\nu}$. Note that $M_{\mu\nu}$ is linear in the fluctuation field $h_{\mu\upsilon}$. In order to achieve this property, it was necessary to introduce the background metric $\bar{g}_{\mu\upsilon}$. Both gauge-fixing and ghost actions depend explicitly on the background metric, as do the source terms, if they are kept linear in the fluctuation field. This implies that the quantum effective action also depends on both metrics: the background metric $\bar{g}_{\mu\nu}$ and the full metric $g_{\mu\nu}(\bar{g}, h)$. Note, however, that the correlation functions of diffeomorphism-invariant operators as well as the solutions to the quantum equations of motion do not depend on the gauge fixing. Hence, they are background-independent as explained below.

它同时包含背景协变导数 $\overline{\nabla}$ 和带全度规 $g_{\mu\nu}$ 联络 (5) 的协变导数 ∇ 。注意 $M_{\mu\nu}$ 对涨落场 $h_{\mu\upsilon}$ 是线性的。要得到这一性质，引入背景度规 $\bar{g}_{\mu\upsilon}$ 是必要的。规范固定作用量和鬼作用量都显式依赖于背景度规，若源项在涨落场中保持线性，源项也同样依赖背景度规。这意味着量子有效作用量同样依赖两个度规: 背景度规 $\bar{g}_{\mu\nu}$ 和全度规 $g_{\mu\nu}(\bar{g}, h)$ 。但需注意，微分同胚不变算符的关联函数以及量子运动方程的解都不依赖规范固定。因此它们如下文所说，是背景无关的。

In summary, we arrive at the generating functional of metric quantum gravity:

综上，我们得到度规量子引力的生成泛函:

$$Z[\bar{g},J] \simeq \int \mathcal{D}\hat{\phi} e^{\mathrm{i}\left(S+S_{\mathrm{gf}}+S_{\mathrm{gh}}+\int \mathrm{d}^4x\sqrt{-\bar{g}}J^a\hat{\phi}_a\right)}. \tag{12}$$

Here, $\hat{\phi}$ includes all fluctuation fields, the graviton fluctuation fields, the ghost fields, and the potential matter fields, and J is composed of the respective currents:

此处 $\hat{\phi}$ 包含所有涨落场: 引力子涨落场、鬼场和可能的物质场，而 J 由对应的流构成:

$$\phi = \left(h_{\mu\nu}, c_\mu, \bar{c}_\mu, \phi_{\mathrm{mat}}\right), \quad \text{and} \quad J = \left(J_{h_{\mu\nu}}, J_{c_\mu}, J_{\bar{c}_\mu}, J_{\mathrm{mat}}\right), \tag{13}$$

where we have dropped the hat. We have not introduced a normalization of the path integral in (12), as it drops out anyway if performing the Legendre transformation to the effective action Γ, the quantum analogue of the classical action. We get

式中我们省略了 hat 标记。我们没有对式 (12) 中的路径积分做归一化，因为在做勒让德变换得到有效作用量 Γ (经典作用量的量子对应) 时，归一化总会被消去。我们得到

$$\Gamma[\bar{g},\phi] = -\mathrm{i}\log Z[\bar{g},J] - \int \mathrm{d}^4x\sqrt{-\bar{g}}J^a\phi_a, \tag{14}$$

where the extremization of the currents is implied. The index a in J^a and ϕ_a comprises all Lorentz, Dirac, gauge group, and further internal indices as well as the species of fields and is summed over in (14). In terms of the superfield and supercurrent in (13), this yields

式中隐含了对流取极值。J^a 和 ϕ_a 中的指标 a 包含所有洛伦兹、狄拉克、规范群及其他内部指标，也包含场的种类指标，在式 (14) 中会被求和。用式 (13) 中的超场和超流表示，可得

$$J^a\phi_a = J_{h_{\mu\nu}}h_{\mu\nu} + J_{c_\mu}c_\mu - \bar{c}_\mu J_{\bar{c}_\mu} + J_{\mathrm{mat}}\phi_{\mathrm{mat}}. \tag{15}$$

So far, we formulated all equations in Lorentzian signature, which is evident from the factors of i in front of the classical action and the minus sign in $\sqrt{-g}$. Within non-perturbative approaches, such as the fRG or lattice, it is almost always necessary to start from a Euclidean formulation of the theory. This is achieved by a Wick rotation where the time component and the four-vector product transform as

到目前为止，我们所有方程都是在洛伦兹号差下表述的，这一点可以从经典作用量前的因子 i 和 $\sqrt{-g}$ 中的负号看出。在非微扰方法中 (例如 fRG 或格点方法)，几乎总是需要从理论的欧几里得表述出发，这通过威克旋转实现，旋转后时间分量和四矢量乘积满足如下变换:

$$t \to -\mathrm{i}t_E, \; x^2 \to -x_E^2. \tag{16}$$

Most notably, this turns the i in the exponentials in (1) and (12) into minus signs, which makes them applicable as probability density for Monte Carlo lattice simulations. In the fRG, this allows for an ordering principle in the momentum-squared for the coarse graining. However, there has been significant recent progress in applying the fRG to Lorentzian settings [81-86]. In the next section, we will set up the Euclidean and Lorentzian fRG equations. Due to the recency of the progress on Lorentzian backgrounds, the majority of results reviewed here are Euclidean and only section ”Lorentzian Quantum Gravity” focuses on Lorentzian results.

最值得注意的是，这会将式 (1) 和式 (12) 指数中的 i 变为负号，使它们可作为蒙特卡罗格点模拟的概率密度使用。在泛函重整化群 (fRG) 中，这为粗粒化过程中的动量平方提供了一个序化原理。不过近年来，将 fRG 应用到洛伦兹情形已经取得了显著进展 [81-86]。在下一节中，我们将建立欧氏和洛伦兹 fRG 方程。由于洛伦兹背景的相关进展出现较晚，本文综述的大多数结果都是欧氏的，仅“洛伦兹量子引力”一节专门讨论洛伦兹结果。

We remark that the physics of a given system is very conveniently expressed in the effective action Γ (see (14)) that by its definition generates one-particle-irreducible correlation functions. Γ has a rather direct connection to physical processes as S -matrix elements can be constructed from Γ in terms of tree-level diagrams with full 1PI vertices and full propagators. Moreover, the gauge-fixed effective action (14) also encodes a diffeomorphism-invariant effective action of one field, the background effective action:

我们注意到，给定系统的物理性质可以非常方便地用有效作用量 Γ 表示 (参见式 (14))，根据定义，有效作用量可以生成单粒子不可约关联函数。Γ 与物理过程有相当直接的联系，因为 S 矩阵元可以由 Γ 通过带完整 1PI 顶点和完整传播子的树图构造出来。此外，规范固定后的有效作用量 (14) 也包含了单一场的微分同胚不变有效作用量，即背景有效作用量:

$$\bar{\Gamma}[g, \phi_{\text{mat}}] = \Gamma[g, \phi]|_{h,c,\bar{c}=0}. \tag{17}$$

Equation (17) only depends on the full metric as well as the matter fields. It is obtained from the full effective action (14) with $h = 0$. Moreover,(17) can also be linked to the S -matrix, even though this relation is less direct. Still, its diffeomorphism invariance allows us to extract many observables directly from the effective action $\bar{\Gamma}$, and it would be most convenient to only work with (17) from the onset. As we shall see, this is only possible within approximation whose applicability has to be assessed very carefully, and such an analysis is rather difficult and has not been done yet. It is the latter intricacies which led to the more direct fluctuation approach discussed in the following three sections: There we first set up the general flow equation approach to metric quantum gravity (see section ”Functional Renormalization Group Approach to Metric Quantum Gravity”) and discuss the ensuing diffeomorphism symmetry constraints (see section ”Background Independence and Symmetry Identities”) and finally the fluctuation approach (see section ”Fluctuation Approach”).

式 (17) 仅依赖于全度量和物质场，它是从含 $h = 0$ 的全有效作用量 (14) 得到的。此外，式 (17) 也可以和 S 矩阵联系起来，尽管这种联系不够直接。尽管如此，它的微分同胚不变性允许我们直接从有效作用量 $\bar{\Gamma}$ 中提取许多可观测量，并且如果从一开始就只使用式 (17) 会非常方便。我们将会看到，这只有在近似框架下才可行，必须非常仔细地评估该近似的适用性，而这类分析相当困难，目前尚未完成。正是这类复杂性催生了接下来三节将要讨论的更直接的涨落方法: 我们首先建立度量量子引力的一般流方程方法 (参见“度量量子引力的泛函重整化群方法”一节)，然后讨论随之而来的微分同胚对称性约束 (参见“背景独立性与对称性恒等式”一节)，最后讨论涨落方法 (参见“涨落方法”一节)。

Functional Renormalization Group Approach to Metric Quantum Gravity

度规量子引力的函数重整化群方法

Based on the quantum field theory approach to metric quantum gravity outlined in the last section, we will now set up fRG equations for gravity; for reviews see [7 − 15, 19, 20] , and for generic fRG reviews, see [21 − 30] .

基于上一节概述的度规量子引力的量子场论方法，我们现在将推导引力的 fRG 方程；综述见 [7 − 15, 19, 20] ，关于 fRG 的通用综述见 [21 − 30] 。

The fRG approach to gravity is based on a successive integrating out of quantum fluctuations. A general regularization of metric quantum gravity is implemented by augmenting the kinetic term of the action S with an IR cutoff term, $S \to S + \Delta S_k$, with

引力的 fRG 方法基于逐次积去量子涨落的框架。我们通过给作用量 S 的动能项添加一个红外截断项 $S \to S + \Delta S_k$ 实现度规量子引力的一般正则化，其中

$$\Delta S_k[\bar{g}, \phi] = \frac{1}{2} \int_{x,y} \phi_a(x) R^{ab}[\bar{g}](x, y) \phi_a(y) , \tag{18}$$

where we have used the abbreviation

我们使用了如下缩写

$$\int_x = \int d^4x \sqrt{\pm \bar{g}(x)}, \tag{19}$$

for the integrals with the volume factor $\sqrt{-\bar{g}}$ in a Lorentzian background and $\sqrt{\bar{g}}$ in a Euclidean one. Common Euclidean regulator classes implement a regularization of covariant momentum modes; more details are provided in section "Euclidean Flow Equation for the Effective Action." In turn, Minkowski regulators either break Lorentz symmetry and only suppress low spatial (covariant) momentum modes or are formulated as spectral cutoffs; more details are provided in section "Lorentzian Flow Equation for the Effective Action." We also note that while these are the common choices, the setup is far more general and interesting other choices are cutoffs for the field amplitude [23] or temporal cutoffs for the time evolution [87].

对应积分: 洛伦兹背景下含体积因子 $\sqrt{-\bar{g}}$ ，欧几里得背景下含体积因子 $\sqrt{\bar{g}}$ 。常见的欧几里得截断类对协变动量模做正则化，更多细节参见“有效作用量的欧几里得流方程”一节。相应地，闵氏截断要么会破坏洛伦兹对称性，仅压低低空间 (协变) 动量模，要么被表述为谱截断；更多细节参见“有效作用量的洛伦兹流方程”一节。另外需要说明，尽管上述是常用选择，但这套框架的适用性远广于此，其他有趣的选择包括场振幅截断 [23] 和时间演化的时间截断 [87]。

Irrespective of its nature of being a momentum, spectral, amplitude, temporal, or other regularization, the cutoff term (18) is conveniently introduced into the generating functional of metric quantum gravity (12), which leads to

无论正则化是动量、谱、振幅还是时间类型，都可以方便地将截断项 (18) 引入度规量子引力的生成泛函 (12)，由此得到

$$Z_k[\bar{g}, J] = \exp\left(-c_g \int_{x,y} \frac{\delta}{\delta J^a(x)} R_k^{ab}(x,y) \frac{\delta}{\delta J^b(y)}\right) Z[\bar{g}, J]. \tag{20}$$

The prefactor c_g depends on the signature of the background metric: we have $c_g = 1$ for a Euclidean background and $c_g = \mathrm{i}$ for a Lorentzian background. Furthermore, we have defined the functional derivative to include a factor of $1/\sqrt{\pm\bar{g}(x)}$, which makes them background-covariant. For example, for a tensor J of rank n , the derivative reads

前置因子 c_g 依赖背景度规的符号: 欧几里得背景下为 $c_g = 1$ ，洛伦兹背景下为 $c_g = \mathrm{i}$ 。此外，我们定义的泛函导数包含一个因子 $1/\sqrt{\pm\bar{g}(x)}$ ，使其满足背景协变性。例如，对秩为 n 的张量 J ，导数形式为

$$\frac{\delta J_{\mu_1\cdots\mu_n}(x)}{\delta J_{\upsilon_1\cdots\upsilon_n}(y)} = \frac{1}{\sqrt{\pm\bar{g}}} \delta(x-y) \delta_{\mu_1}^{(\upsilon_1\cdots\delta_{\mu_n}^{\upsilon_n})}. \tag{21}$$

This notation was also used in [31].

文献 [31] 也使用了该记号。

Euclidean Flow Equation for the Effective Action

有效作用量的欧几里得流方程

Here we review the standard fRG approach to metric quantum gravity: as mentioned above, the commonly used regulator function is a function of background covariant derivatives and hence ultra-local in position space, $R^{ab}(x,y) \propto \delta(x-y)$. Here we consider such cutoffs and elucidate their use at the example of Euclidean quantum gravity. The implementation of such a covariant momentum cutoff necessitates the choice of a background metric $\bar{g}$, and the covariant momenta are those related to the covariant Laplacian in the background metric, $\Delta_{\bar{g}}$, with the spectral values $p_{\bar{g}}^2$.

我们在此回顾度规量子引力的标准 fRG 方法: 如上所述，常用调节函数是背景协变导数的函数，因此在位置空间是超局域的，$R^{ab}(x,y) \propto \delta(x-y)$ 。我们在此考虑这类截断，并以欧几里得量子引力为例阐明其应用。实现这类协变动量截断需要选定背景度规 $\bar{g}$ ，协变动量对应背景度规下的协变拉普拉斯算符，$\Delta_{\bar{g}}$ ，谱值为 $p_{\bar{g}}^2$ 。

Most of the computations in the fluctuation approach discussed here are done in a flat background with the notable exception of the computations in [88, 89] . In these works, the theory was expanded about a curved background $\bar{g}_{\mu\nu} \neq \delta_{\mu\nu}$, which allows us to compute background-metric-dependent couplings and also provides direct access to the diffeomorphism-invariant background effective action. In a curved background, the regulator depends on the Laplace-Beltrami operator in this background as well as background curvature terms. The background can be chosen such that it is the dynamical solution of the full quantum equation of motion in which case the flow equation takes into account fluctuations about this background. While this may facilitate the convergence of expansion schemes, it is by no means a necessary choice.

本文讨论的涨落方法中，除 [88, 89] 中的计算外，绝大多数计算都是在平直背景下完成的。这些工作围绕弯曲背景 $\bar{g}_{\mu\nu} \neq \delta_{\mu\nu}$ 对理论做展开，这使得我们可以计算依赖背景度规的耦合，也可以直接得到微分同胚不变的背景有效作用量。在弯曲背景下，调节子依赖于该背景下的拉普拉斯-贝尔特拉米算符以及背景曲率项。背景可以选为全量子运动方程的动力学解，这种情况下流方程会自动计入该背景周围的涨落。虽然这可能有助于提升展开方案的收敛性，它绝非必需的选择。

In the following, we concentrate on a flat Euclidean background:

在下文中，我们聚焦于平直欧几里得背景:

$$\bar{g}_{\mu\upsilon} = \delta_{\mu\upsilon}. \tag{22}$$

With (22), the eigenvalues of the Laplacian are just momentum-squared, p^2 , and the regulator suppresses the IR-momentum modes with $p^2 \lesssim k^2$. In turn, UV-momentum modes with $p^2 \gtrsim k^2$ propagate freely, and the generating functional includes all quantum contributions generated by these modes. Then, one RG step with $k \to k - \Delta k$ constitutes the integration of momentum modes $p^2 \approx k^2$.

根据 (22) 式，拉普拉斯算符的本征值恰好是动量平方，p^2 ，调节子会压制满足 $p^2 \lesssim k^2$ 的红外动量模式。相应地，满足 $p^2 \gtrsim k^2$ 的紫外动量模式可以自由传播，生成泛函包含了这些模式产生的所有量子贡献。随后，带 $k \to k - \Delta k$ 的一次重整化群步骤就完成了对动量模式 $p^2 \approx k^2$ 的积分。

It is convenient to write the regulator R_k in terms of the classical or quantum dispersion of the field at hand:

将调节子 R_k 用当前场的经典或量子色散来表示会更为方便:

$$R_k^{ab}(p) = T_k^{ab}(p)\, r_k(x), \text{ with } x = \frac{p^2}{k^2}, \tag{23}$$

where momentum-squared is counted in cutoff units. In these units, the IR regime is given by $x \lesssim 1$ and the UV regime by $x \gtrsim 1$. The tensor part T_k^{ab} of the regulator is proportional to the classical or quantum dispersion of the field. Classically, it is the second derivative of the action with respect to the fields ϕ^a and ϕ^b , i.e., $T_k^{ab}(p) = \left(S^{(2)}\right)^{ab}(p)$. It carries the kinetic information about the field whose propagation is regularized. In turn, the dimensionless shape function r_k specifies how the propagation is regularized. The shape function r_k has to be chosen such that the IR suppression of momentum modes as well as the UV decay of the regularization is guaranteed. In the flow equation approach to Lorentzian quantum gravity setup in [82], we will employ a slightly different strategy in regularizing the spectral UV modes; see section ”Lorentzian Flow Equation for the Effective Action.” For Euclidean quantum gravity, the regularization of UV and IR modes leads to the following asymptotics of the regulator shape function:

此时动量平方以截断单位计数。在该单位制下，红外区对应 $x \lesssim 1$ ，紫外区对应 $x \gtrsim 1$ 。调节子的张量部分 T_k^{ab} 正比于场的经典或量子色散。经典意义上，它是作用量对场 ϕ^a 和 ϕ^b 的二阶导数，即 $T_k^{ab}(p) = \left(S^{(2)}\right)^{ab}(p)$ 。它承载了被正则化传播的场的动能信息。相应地，无量纲形状函数 r_k 指定了传播的正则化方式。形状函数 r_k 的选择需要满足两个条件: 保证动量模式的红外压制和正则化的紫外衰减。在文献 [82] 建立的洛伦兹量子引力流方程方案中，我们对谱紫外模式采用了略有不同的正则化策略；参见“有效作用量的洛伦兹流方程”一节。对欧几里得量子引力而言，紫外和红外模式的正则化给出调节子形状函数的如下渐近行为:

$$\lim_{x\to 0} r_k(x) \to \infty, \ \lim_{x\to\infty} x^{d/2} r_k(x) = 0, \tag{24}$$

where d is the dimension of space-time. The first limit in (24) guarantees the IR suppression of momentum modes and also entails that for $k \to \infty$ all momentum modes are suppressed and the theory approaches the UV-scaling regime. For asymptotically free theories, this is the classical theory, while for asymptotically safe theories, this is the nontrivial UV theory. The second limit in (24) guarantees that the UV behavior of the theory is unchanged by the IR regularization. Moreover, the decay with more than $x^{d/2}$ guarantees that a change of the IR RG scale does not require an adaptation of the UV renormalisation. This limit also has another implication: for $k \to 0$ the limit holds for all momenta and the cutoff is removed from the theory.

其中 d 是时空的维度。式 (24) 中的第一个极限保证了动量模的红外压制，同时也意味着当 $k \to \infty$ 时，所有动量模都被压制，理论趋近于紫外标度区。对于渐近自由理论，这就是经典理论；而对于渐近安全理论，这是非平凡的紫外理论。式 (24) 中的第二个极限保证了理论的紫外行为不会被红外正规化改变。此外，衰减快于 $x^{d/2}$ 保证了红外重整化群尺度的变化不需要调整紫外重整化。这个极限还有另一层含义: 当 $k \to 0$ 时，该极限对所有动量都成立，截除项从理论中移除。

The flow equation for the scale-dependent effective action $\Gamma_k[\bar{g}, \phi]$ is then given by [2, 54, 55]

依赖于尺度的有效作用量 $\Gamma_k[\bar{g}, \phi]$ 的流方程由 [2, 54, 55] 给出

$$\partial_t \Gamma_k[\bar{g}, \phi] = \frac{1}{2} \operatorname{Tr} G_k[\bar{g}, \phi]\, \partial_t R_k, \tag{25}$$

where $t = \log(k/k_0)$ with some reference scale k_0 , and $G_k[\bar{g}, \phi]$ is the full field-dependent propagator:

其中 $t = \log(k/k_0)$ 带有参考标度 k_0 ， $G_k[\bar{g}, \phi]$ 是依赖场的全传播子:

$$G_k[\bar{g}, \phi] = \frac{1}{\Gamma_k^{(0,2)}[\bar{g}, \phi] + R_k}, \tag{26}$$

and the trace sums over space-time, internal and gauge group indices, and species of fields, including the symplectic metric for fermions. In (26), we have used the notation $\Gamma_k^{(0,2)}$ for the second derivative of the effective action with respect to the dynamical fluctuation fields ϕ . In general, we use the following notation for derivatives:

迹对时空、内禀和规范群指标、场种类求和，包含费米子的辛度量。式 (26) 中，我们用记号 $\Gamma_k^{(0,2)}$ 表示有效作用量对动力学涨落场 ϕ 的二阶导数。一般而言，我们对导数使用如下记号:

$$\Gamma^{(n,m)}_{k,\bar{g}^n\phi_{a_1}\cdots\phi_{a_m}}[\bar{g},\phi](\mathbf{x}_{n+m}) = \frac{\delta}{\delta\bar{g}(x_1)}\cdots\frac{\delta}{\delta\bar{g}(x_n)}$$

$$\times\frac{\delta}{\delta\phi_{a_1}(x_{n+1})}\cdots\frac{\delta}{\delta\phi_{a_m}(x_{n+m})}\Gamma_k[\bar{g},\phi]\,, \tag{27}$$

for general functionals of $\bar{g}$ and ϕ . On the left-hand side, we used the abbreviation

适用于 $\bar{g}$ 和 ϕ 的任意泛函。在左侧，我们使用了缩写

$$\mathbf{x}_m = (x_1, \ldots, x_m)\,, \tag{28}$$

which we will use in straight analogy for sets for four-momenta, $\mathbf{p}_m = (p_1, \ldots, p_m)$.

我们将按照完全类似的方式，把该缩写用于四动量集合 $\mathbf{p}_m = (p_1, \ldots, p_m)$。

In (27), the subscript indicates the k -dependence and the fields indicate which field derivative is taken and in which order. The latter is important as the fluctuation field includes also fermionic Grassmannian fields such as the gravity ghosts. Moreover, we shall also use the abbreviations

式 (27) 中，下标表示对 k 的依赖，场指标指明对哪个场求导以及求导顺序。后者很重要，因为涨落场也包含费米性格拉斯曼场，例如引力鬼场。此外，我们还会使用如下缩写

$$\Gamma_k^{(n,m)}, \Gamma_k^{(n,\phi_{a_1}\cdots\phi_{a_m})} = \Gamma^{(n,m)}_{k,\bar{g}^n\phi_{a_1}\cdots\phi_{a_m}}, \; \Gamma_k^{\phi_{a_1}\cdots\phi_{a_m}} = \Gamma_k^{(0,\phi_{a_1}\cdots\phi_{a_m})}, \tag{29}$$

for (27) with or without the subscript $_k$ for the sake of brevity. This concludes our derivation of the functional flow equation in metric quantum gravity.

为了简洁，用于式 (27) 时可带或不带下标 $_k$。至此我们完成了度量量子引力中泛函流方程的推导。

Lorentzian Flow Equation for the Effective Action

有效作用量的洛伦兹流方程

In this section, we review the spectral fRG approach to Lorentzian quantum gravity that has been put forward in [82]. It constitutes the first direct Lorentzian fRG approach and was used for the computation of the spectral function of the graviton propagator.

本节中，我们将评述文献 [82] 提出的洛伦兹量子引力谱 fRG 方法。这是首个直接的洛伦兹 fRG 方法，已被用于计算引力子传播子的谱函数。

The standard Euclidean setting allows for an ordering of fluctuations according to the size of the Euclidean momentum-squared, p^2 , or more generally in terms of some covariant spectral value as the spectral values of the Laplace-Beltrami operator. The respective numerical results have to be Wick-rotated in order to use them for Lorentzian quantum gravity. This is already a challenging task in a standard quantum field theory, where the Wick rotation is mathematically well-defined. There, the intricacy is solely given by the task of reconstructing real-time data from imaginary data, which is an ill-conditioned problem. Moreover, in gauge theories, it is further complicated due to the fact that at least the gauge field is not a physical field and its spectral representation is at stake; for a respective discussion, see [90]. In asymptotically safe quantum gravity, the reconstruction problem is softened by the close to perturbativeness of asymptotically safe gravity; see [91] for a brief description and respective references. This has been used for the reconstruction of the graviton propagator from its Euclidean data in [92].

标准欧几里得框架允许按照欧几里得动量平方的大小对涨落排序，p^2，或者更一般地，按照协变谱值 (例如拉普拉斯-贝尔特拉米算子的谱值) 排序。想要将所得数值结果用于洛伦兹量子引力，必须先做威克旋转。即使在威克旋转数学定义良好的标准量子场论中，这也是一项极具挑战性的任务。其中，难点仅在于如何从虚数数据重建实时数据，而这本身是一个病态问题。此外，在规范理论中，该问题进一步复杂化: 至少规范场不是物理场，其谱表示本身就存在争议，相关讨论参见文献 [90]。在渐近安全量子引力中，由于渐近安全引力接近微扰性，重建问题有所缓解，简要介绍和相关参考文献参见 [91]。文献 [92] 已利用这一点从欧几里得数据重建了引力子传播子。

Evidently, a direct Lorentzian renormalization group approach is much wanted and has been set up in quantum gravity in [82] within the spectral fRG approach [83] based on the general spectral functional approach put forward in [81]. The spectral fRG utilizes the spectral representation of correlation functions and most prominently the propagators $G_{\phi_a\phi_b}(p)$. Here we briefly present the key idea; for more details, we refer the reader to [82, 83]. For the discussion of spectral representations, we use the Källén-Lehmann spectral representation of a scalar Euclidean propagator with

显然，人们迫切需要直接的洛伦兹重整化群方法，该方法已由文献 [82] 在谱 fRG 框架 [83] 中构建完成，其基础是文献 [81] 提出的一般谱泛函方法。谱 fRG 利用了关联函数、最突出是传播子 $G_{\phi_a\phi_b}(p)$ 的谱表示。本文简要介绍其核心思想，更多细节请读者参阅文献 [82, 83]。为讨论谱表示，我们使用标量欧几里得传播子的卡伦-莱曼谱表示

$$G\left(p_0, |\vec{p}|\right) = \int_{0_-}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \frac{\lambda\rho\left(\lambda, |\vec{p}|\right)}{\lambda^2 + p_0^2}, \tag{30}$$

where $p_0 \in \mathbb{R}$ singles out the Euclidean frequency axis. However, the frequency can take any complex value, $p_0 \in \mathbb{C}$, including Lorentzian frequencies ω with $p_0 = \mathrm{i}\omega$. The lower bound of the spectral integral, 0_-, takes into account that for massless particles, the spectral function contains a delta function at vanishing spectral values, $\delta(\lambda)$. Equation (30) can be inserted into loop diagrams and, upon integration of the analytic frequency dependence, one is led to numerically accessible spectral and spatial momentum integrals without poles. This is the key idea in a nutshell.

其中 $p_0 \in \mathbb{R}$ 单独标记欧几里得频率轴。不过频率可以取任意复数值 $p_0 \in \mathbb{C}$，包括满足 $p_0 = \mathrm{i}\omega$ 的洛伦兹频率 ω。谱积分的下界 0_- 考虑了如下情况: 对于无质量粒子，谱函数在零谱值 $\delta(\lambda)$ 处包含一个 δ 函数。将式 (30) 代入圈图，对解析频率依赖积分后，我们就得到了可数值计算、不含极点的谱动量积分与空间动量积分。简言之，这就是其核心思想。

It is left to define the class of cutoffs that preserve the spectral representation (30). Moreover, we would like to maintain causality and unitarity, Lorentz invariance, and finiteness of the flow. The causality and unitarity constraint is tightly related to the existence of the spectral representation and constrains the position of poles in the complex frequency plane. For this discussion, we restrict ourselves to the propagator of a scalar field. This argument is readily generalized if writing a given propagator as the dimensionless tensor structure times the scalar part of the propagator. In contradistinction to (23), we use the parametrization

接下来需要定义保持谱表示 (30) 的截断类。此外，我们希望保留因果性、幺正性、洛伦兹不变性以及流的有限性。因果性与幺正性约束和谱表示的存在密切相关，它限制了复频率平面上极点的位置。在下文讨论中，我们仅关注标量场的传播子。只要将任意给定传播子写为无量纲张量结构乘以传播子的标量部分，该论证就可以直接推广。与 (23) 不同，我们使用如下参数化

$$R_k(p) = k^2 r(x)\,, \tag{31}$$

also used in [83], where the spectral RG is introduced. Equation (31) entails that the shape function $r(x)$ carries the whole momentum dependence of the regulator and the prefactor k^2 carries the dimension. Then, the causality and unitarity requirements only constrain the shape function, and this constraint is satisfied for regulators with shape functions r_k defined in (31) with

该形式也用于文献 [83]，其中介绍了谱 RG。式 (31) 表明，形状函数 $r(x)$ 承载了调节器的全部动量依赖，而前置因子 k^2 承载量纲。因此，因果性与幺正性要求仅约束形状函数，对于满足下述条件、由 (31) 定义的形状函数 r_k ，该约束成立:

$$r_k = r_k\left(x_{\vec{p}}\right), \text{ with } x_{\vec{p}} = \frac{\vec{p}^2}{k^2}. \tag{32}$$

However, (32) breaks Lorentz invariance. The Lorentz invariance constraint is only satisfied for regulators with shape functions r_k depending on the Lorentz invariant four-momentum-squared:

然而，(32) 破坏了洛伦兹不变性。只有当形状函数 r_k 依赖于洛伦兹不变的四动量平方时，才满足洛伦兹不变性约束:

$$r_k = r_k\left(x_p\right), \text{ with } x_p = \frac{p^2}{k^2}, \tag{33}$$

which in turn do not satisfy the causality constraint. Finally, both shape functions can guarantee the finiteness of the flow if they satisfy

而这类形状函数不满足因果性约束。最后，若两类形状函数满足下式，就都可以保证流的有限性:

$$\lim_{x\to 0} r_k(x) > 0, \ \lim_{x\to\infty} x^{d/2-1} r_k(x) = 0, \tag{34}$$

the analogue of the limits (24) for the parametrization (23).

即参数化 (23) 对应极限 (24) 的类比形式。

The different demands for the shape function $r(x)$ in (31) are depicted in Fig. 1, and a detailed discussion can be found in [83]. To date, the construction of a regulator with all the above properties is lacking. As the spectral fRG approach rests upon the existence of a spectral representation, or at least is facilitated by its prevalence, we rely on regulators that fulfill the causality constraint. Still, we note that Lorentz-invariant regulators with momentum argument (33) can be constructed, which also satisfy (34). The respective real-time flows are then based on the Keldysh contour and satisfy the causality constraint in a given frequency domain; see [93].

式 (31) 中形状函数 $r(x)$ 的不同要求如图 1 所示，详细讨论可见文献 [83]。目前，尚未构建出满足上述全部性质的调节子。由于谱 fRG 方法依赖谱表示的存在 (至少谱表示的通用性为该方法提供了便利)，我们使用满足因果性约束的调节子。我们注意到，仍可构造带有动量宗量 (33) 的洛伦兹不变调节子，这类调节子同样满足式 (34)；对应的实时流基于凯尔迪什轮廓，在给定频域内满足因果性约束，参见文献 [93]。

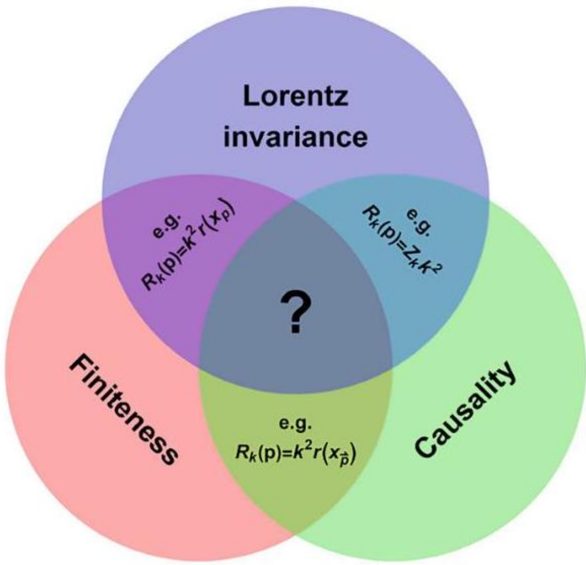
For general regulators with (32), the price to pay is the breaking of Lorentz invariance, which is restored in the limit $k \to 0$. The only regulator that preserves causality and Lorentz invariance without any restriction is the Callan-Symanzik (CS) cutoff

对于满足式 (32) 的一般调节子，需要付出的代价是洛伦兹不变性破缺，该不变性可在极限 $k \to 0$ 下恢复。唯一不受限制同时保留因果性与洛伦兹不变性的调节子是卡伦-西曼齐克 (CS) 截断

$$R_k = Z_\phi k^2. \tag{35}$$

Fig. 1 Sketch of the competing requirements for regulators: finiteness of the flow, Lorentz invariance, and causality of regulators. Examples of regulators with two of the properties are given. A fully systematic construction of regulators with all three properties in the flow is lacking to date. (Figure adapted from [83])

图 1 调节子竞争要求示意图: 流有限、洛伦兹不变性、调节子因果性。图中给出了满足其中两个性质的调节子示例。目前仍未系统构建出在流中同时满足全部三个性质的调节子。(图改编自文献 [83])



These properties make the CS cutoff best suited to extract spectral data. The price to pay is that the flow requires additional renormalization because the standard UV divergences and counter-terms resurface [94]. These local divergences of the flow must be absorbed in the parameters of the cutoff-dependent effective action. The CS regulator was later also employed in similar Lorentzian flow equations in algebraic QFT [85, 86].

上述性质使得 CS 截断最适合提取谱数据。需要付出的代价是，该流需要额外重整化，因为标准紫外发散和 counter-term 重新出现 [94]。流的这些局部发散必须被依赖截断的有效作用量的参数吸收。CS 调节子后来也被用于代数量子场论中类似的洛伦兹流方程 [85, 86]。

Using the CS cutoff in the general flow equation (25) leads us to the functional CS equation, which is first put down in [94]. Indeed, it constitutes the first functional flow equation. Typically, it is renormalized by order in perturbation theory with the introduction of renormalized fields, couplings, and masses. This procedure cannot be applied within non-perturbative applications, and in [83] a general functional flow equation for the effective action Γ_k with flowing renormalization has been derived. It reads for both Euclidean and Lorentzian signatures:

在一般流方程 (25) 中使用 CS 截断，我们得到了最初在文献 [94] 中提出的泛函 CS 方程。事实上，它是首个泛函流方程。通常，该方程通过引入重整化场、耦合和质量按微扰论阶数进行重整化。该过程无法应用于非微扰场景，文献 [83] 推导出了带流动重整化、针对有效作用量 Γ_k 的一般泛函流方程，该方程对欧几里得和洛伦兹号差均成立，形式如下:

$$\partial_t \Gamma_k [\phi] = \frac{1}{2} \operatorname{Tr} G_k [\phi] \, \partial_t R_k - \partial_t S_{\mathrm{ct},k} [\phi] \, , \tag{36}$$

with the local flowing counter-term action $S_{\mathrm{ct},k}$. Importantly,(36) allows for a flowing renormalization, which is readjusted at each RG step. In (36), the CS limit for momentum cutoffs with (32) and (33) was taken in a manifestly finite way as the flowing counter-term action guarantees finiteness of the flow. We emphasize that in the CS limit, only the combination of the two terms on the right-hand side of (36) is finite, and strictly speaking (36) only provides the structure of the flow.

式中包含局部流动 counter-term 作用量 $S_{\mathrm{ct},k}$ 。重要的是，式 (36) 支持流动重整化，可在每个 RG 步骤重新调整。式 (36) 中，带 (32) 和 (33) 的动量截断的 CS 极限以明显有限的方式取得，因为流动 counter-term 作用量保证了流的有限性。我们强调，在 CS 极限下，仅式 (36) 右侧两项的组合是有限的，严格来说式 (36) 仅给出了流的结构。

Importantly, the analytic nature of the momentum integrals in a spectral functional approach allows us to also define and use spectral dimensional regularization non-perturbatively and hence preserve gauge and diffeomorphism invariance within a non-perturbative numerical setup. Of course, also other regularization schemes such as the spectral BHPZ regularization can be used; for a detailed discussion, see [81,83], and for applications see [81,95-98]. In the explicit numerical computations of the graviton propagator in [82], spectral dimensional regularization was used since it respects diffeomorphism invariance. More details and results of the spectral fRG approach to asymptotically safe metric quantum gravity are presented in section ”Lorentzian Quantum Gravity.”

重要的是，谱泛函方法中动量积分的解析性质允许我们在非微扰层面定义和使用谱维正规化，从而在非微扰数值框架中保留规范不变性和微分同胚不变性。当然，也可使用其他正规化方案，例如谱 BHPZ 正规化；详细讨论参见文献 [81,83]，应用案例参见文献 [81,95-98]。文献 [82] 中对引力子传播子的显式数值计算，使用了谱维正规化，因为该方案满足微分同胚不变性。关于渐近安全度规量子引力的谱 fRG 方法，更多细节与结果参见“洛伦兹量子引力”一节。

Flow Equations of Correlation Functions

关联函数的流方程

The functional flow equations, (25) and (36), for the effective action cannot be solved in a closed form. In most quantum field theories, the effective action has to be expanded in a systematic expansion scheme, and the flow is then solved in a given order of this expansion scheme. Here, we discuss this approach in terms of an expansion in terms of n, m -point correlation functions, where n refers to background derivatives and m to fluctuation derivatives; see (27). They constitute the expansion coefficients of an expansion of the effective action $\Gamma_k[\bar{g}, \phi]$ in terms of powers of the fluctuation field and the background metric. The effective action takes the form

有效作用量的泛函流方程，即式 (25) 和式 (36)，无法得到闭式解。在大多数量子场论中，必须将有效作用量纳入系统展开方案中进行展开，再在该展开方案的给定阶数下求解流方程。我们在此基于 n, m 点关联函数的展开讨论该方法，其中 n 对应背景导数，m 对应涨落导数；参见式 (27)。它们构成了有效作用量 $\Gamma_k[\bar{g}, \phi]$ 按涨落场和背景幂次展开的展开系数。有效作用量形式如下

$$\Gamma_k[\bar{g}, \phi] = \sum_{m=0}^{\infty} \int_{\mathbf{x}_m} \Gamma_k^{\phi_{a_1} \cdots \phi_{a_m}}[\bar{g}, 0](\mathbf{x}_m)\, \phi_{a_1}(x_1) \cdots \phi_{a_m}(x_m)\,. \tag{37}$$

In (37), we have used the abbreviation in (29), and a contraction of the indices a_i as in (15) is implied. Moreover, for the sake of simplicity, we have restricted ourselves to an expansion about $\phi = 0$, but the expression is readily extended to one about a general background $\bar{\phi}$. We emphasize that the expansion coefficients $\Gamma_k^{\phi_{a_1} \cdots \phi_{a_m}}[\bar{g}, 0]$ in (37) also comprise all $\Gamma_k^{(n,m)}$ upon n further derivatives with respect to $\bar{g}$.

在式 (37) 中，我们使用了式 (29) 的缩写，且默认按式 (15) 对指标 a_i 做缩并。此外，为简化推导，我们将展开限定在围绕 $\phi = 0$ 的范围内，但该表达式可以很容易地推广到围绕一般背景 $\bar{\phi}$ 展开的情况。我们强调，式 (37) 中的展开系数 $\Gamma_k^{\phi_{a_1} \cdots \phi_{a_m}}[\bar{g}, 0]$ 已经包含了对 $\bar{g}$ 进一步求导后得到的所有 $\Gamma_k^{(n,m)}$，即 n 。

The set of derivatives $\Gamma_k^{(n,m)}$ is the set of all one-particle irreducible parts of correlation functions of the fluctuation as well as the background field. As such, it provides the full information about metric quantum gravity. More precisely, this information is comprised in the dressings of the $\Gamma_k^{(n,m)}$, which are evaluated on the equations of motion or another expansion point. We expand the correlation functions $\Gamma_k^{(n,m)}$ in complete tensor bases:

导数集合 $\Gamma_k^{(n,m)}$ 是涨落和背景场所有单粒子不可约关联函数部分的集合，因此它包含了度规量子引力的全部信息。更准确地说，这些信息包含在 $\Gamma_k^{(n,m)}$ 的修饰项中，修饰项在运动方程或其他展开点上计算。我们将关联函数 $\Gamma_k^{(n,m)}$ 按完备张量基展开:

$$\Gamma_k^{(n,m)}[\bar{g},0] = \sum_i^{N_{n,m}} \lambda_{n,m}^{(i)} \mathcal{T}_{n,m}^{(i)}, \tag{38}$$

where $i = 1, \ldots, N_{n,m}$ labels the tensors in the basis $\left\{\mathcal{T}_{n,m}^{(i)}\right\}$ and $N_{n,m}$ is its size, which increases rapidly with n and m . The whole k -dependence of $\Gamma_k^{(n,m)}$ is carried by the dressings $\lambda_{n,m}$, while the tensor basis is typically kept k -independent. Note also that in (38), we have used the short-hand notations (29) in order to keep things simple. For a given m , there are many different $\Gamma^{(n,m)}$ with different bases and different tensors and dressings:

其中 $i = 1, \ldots, N_{n,m}$ 标记基 $\left\{\mathcal{T}_{n,m}^{(i)}\right\}$ 中的张量，$N_{n,m}$ 是基的大小，其随 n 和 m 迅速增大。$\Gamma_k^{(n,m)}$ 对 k 的全部依赖都由修饰项 $\lambda_{n,m}$ 承担，而张量基通常与 k 无关。另请注意，在式 (38) 中，我们为简化表述使用了式 (29) 的简写记号。对于给定的 m，存在许多不同的 $\Gamma^{(n,m)}$，它们拥有不同的基、不同的张量和不同的修饰项:

$$\mathcal{T}_{n,m}^{(i)} \to \mathcal{T}_{n,\phi_{s_1}\cdots\phi_{s_m}}^{(i)}(\mathbf{x}_{n+m}), \; \lambda_{n,m}^{(i)} \to \lambda_{n,\phi_{s_1}\cdots\phi_{s_m}}^{(i)}(\mathbf{x}_{n+m}). \tag{39}$$

In general, the tensors and dressings in (39) depend on the chosen background metric $\bar{g}$ and for $\overline{\phi} \neq 0$, also on the chosen fluctuation field. For $\overline{\phi} = 0$ and $\bar{g} = \eta$, the Fourier transformation can be readily performed, leading to

一般而言，式 (39) 中的张量和修饰项依赖于所选的背景度规 $\bar{g}$，对于 $\overline{\phi} \neq 0$ 还依赖于所选的涨落场。对于 $\overline{\phi} = 0$ 和 $\bar{g} = \eta$，可以很容易地做傅里叶变换，得到

$$\mathcal{T}_{n,\phi_{s_1}\cdots\phi_{s_m}}^{(i)}(\mathbf{p}_{n+m}), \; \lambda_{n,\phi_{s_1}\cdots\phi_{s_m}}^{(i)}(\mathbf{p}_{n+m}). \tag{40}$$

The tensor basis is typically chosen such that the tensors are regular in momentum space. Importantly, this allows us to distinguish parametric singularities in the dressings from physical ones. While this is seemingly relatively trivial, physical irregularities in the dressings can have far-reaching consequences, a prominent example being QCD, where irregular vertices are required for the presence of confinement in a covariant gauge such as also used in quantum gravity; see, e.g., [99] and literature therein. In short, the choice of a well-defined tensor basis may be of eminent importance for unraveling the underlying physics of the system at hand.

张量基通常选得让张量在动量空间中是正则的。重要的是，这能让我们区分修饰项中的参数奇点与物理奇点。虽然这看起来十分平凡，但修饰项中的物理不规则性能带来影响深远的结果，一个突出的例子就是量子色动力学: 在协变规范中 (该规范也被用于量子引力)，禁闭的存在要求顶点具有不规则性，参见例如文献 [99] 及其中引述的研究。简而言之，选择定义良好的张量基对于揭示当前系统的底层物理至关重要。

The functional flow equations, (25) and (36), allow us to compute the scale dependence of the effective action in terms of the correlation functions $\Gamma_k^{(n,m)}$ or rather the dressings $\lambda_{n,m}^{(i)}$. To begin with, the flow of the $\Gamma_k^{(n,m)}$ is obtained from the flow of $\Gamma_k[\bar{g},\phi]$ by applying n derivatives with respect to $\bar{g}$ and m derivatives with respect to ϕ . This leads us to

泛函流方程，即式 (25) 和式 (36)，允许我们通过关联函数 $\Gamma_k^{(n,m)}$ 更准确地说是修饰项 $\lambda_{n,m}^{(i)}$ 计算有效作用量的标度依赖性。首先，$\Gamma_k^{(n,m)}$ 的流可通过对 $\Gamma_k[\bar{g},\phi]$ 的流分别取关于 $\bar{g}$ 的 n 阶导数和关于 ϕ 的 m 阶导数得到，由此我们得到

$$\partial_t \Gamma_k^{(n,m)}[\bar{g},\phi] = \frac{\delta^{n+m}\partial_t \Gamma_k[\bar{g},\phi]}{\delta^n \bar{g}\delta^m \phi} \equiv \mathrm{Flow}^{(n,m)}[\bar{g},\phi], \tag{41}$$

where we have used the k -independence of the fields. Flow $^{(n,m)}$ stands for the diagrams generated from the field derivatives on the right-hand side of (25).

其中我们利用了场不依赖于 k 的性质。流 $^{(n,m)}$ 代表由式 (25) 右侧场求导生成的图。

Equation (41) is a functional equation that holds for general background metrics and fluctuation fields. It hence offers a simple way to generate the diagrams on the right-hand side of the flow equation by successively applying field derivatives and using the rules

式 (41) 是适用于任意背景度规和涨落场的泛函方程，因此它提供了一种简便方法: 通过逐次做场求导并应用下述规则生成流方程右侧的图

$$\frac{\delta}{\delta\phi_b}\Gamma^{\phi_{a_1}\cdots\phi_{a_m}} = \Gamma^{\phi_b\phi_{a_1}\cdots\phi_{a_m}} \tag{42a}$$

and

和

$$\frac{\delta}{\delta\phi_b}G_{\phi_{a_1}\phi_{a_2}} = (-1)^{a_1 b}G_{\phi_{a_1}\phi_c}\Gamma^{\phi_c\phi_b\phi_d}G_{\phi_d\phi_{a_2}}, \tag{42b}$$

where $(-1)^{a_1 b} = -1$ for ϕ_{a_1} and ϕ_b both being fermionic and $(-1)^{a_1 b} = 1$ for all other cases. As an explicit example, we show in Fig. 2 the diagrammatic flows of $\Gamma^{(0,n)}$ for the graviton and ghost fields up to $n = 4$, which can be readily derived from the rules (42). Equation (42) extends readily to derivatives with respect to the background metric and allows us to iteratively derive the flow equations (41).

其中当 ϕ_{a_1} 和 ϕ_b 均为费米子时取 $(-1)^{a_1 b} = -1$ ，所有其他情况取 $(-1)^{a_1 b} = 1$ 。作为显例，我们在图 2 中给出了引力子和鬼场直至 $n = 4$ 阶的 $\Gamma^{(0,n)}$ 图流，它可以直接通过规则 (42) 推导出。式 (42) 可以直接推广到关于背景度规的求导，允许我们迭代推导出流方程 (41)。

The right-hand side of the flows of general correlation functions (41) only contains correlation functions with at least two fluctuation fields:

一般关联函数流 (41) 的右侧仅包含至少含两个涨落场的关联函数:

$$\partial_t \Gamma_k^{(n,m)} = \text{Flow}^{(n,m)}\left[\bar{g}, \left\{\Gamma_k^{(i,j)}\right\}\right], \quad \text{with} \ \ 0 \leq i \leq n \text{ and } 2 \leq j \leq m+2;$$

(43)

see also Fig. 2 as explicit example. This entails that correlation functions with higher orders in $\bar{g}$ depend on lower order ones in $\bar{g}$. Moreover, the flow of a correlation function $\Gamma_k^{(n,m)}$ depends on the correlation functions $\Gamma_k^{(i,m+1)}, \Gamma_k^{(i,m+2)}$ with $i \leq n$: the system does not close on a given order in the fluctuation field, while it closes on a given order of the background field. Note that the origin of these properties is the dynamic nature of the fluctuation field and the auxiliary nature of the background field. In short, the correlation functions of the background field can only be computed if that of the fluctuation field are known.

另见图 2 的具体示例。这意味着 $\bar{g}$ 更高阶的关联函数依赖于 $\bar{g}$ 更低阶的关联函数。此外，关联函数 $\Gamma_k^{(n,m)}$ 的流依赖于满足 $i \leq n$ 的关联函数 $\Gamma_k^{(i,m+1)}, \Gamma_k^{(i,m+2)}$: 系统在涨落场的给定阶不封闭，但在背景场的给定阶封闭。请注意这些性质的起源是涨落场的动力学性质和背景场的辅助性质。简而言之，只有已知涨落场的关联函数，才能计算背景场的关联函数。

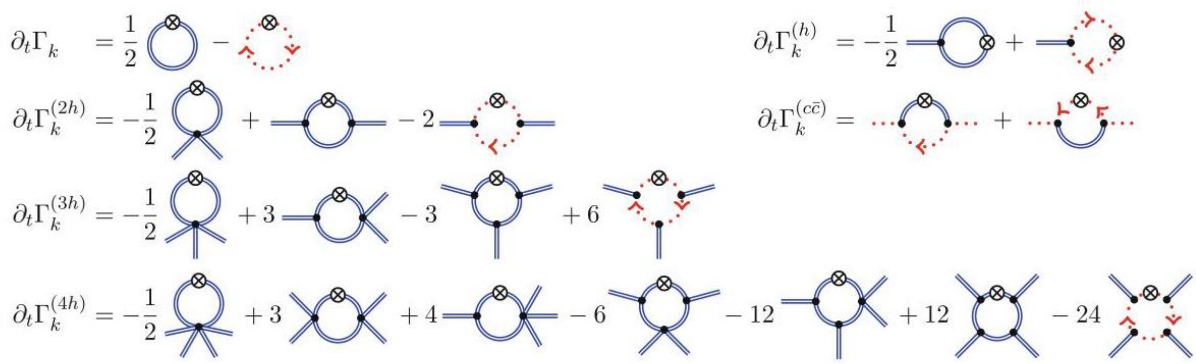


Fig. 2 We display the diagrammatic flow of the n -point correlation functions. Blue double lines represent the fluctuation graviton propagators, red dotted lines stand for ghost propagators, while filled and crossed circles display dressed vertices and regulator insertions, respectively

图 2 我们展示了 n 点关联函数的图流。蓝色双线代表涨落引力子传播子，红色虚线代表鬼传播子，实心圆和叉圆分别代表修饰顶点和正规化调节器插入项

In turn, the fluctuation correlation functions $\Gamma^{(0,n)}\left[\bar{g}\bar{\phi}\right]$ constitute a closed system of flows for all backgrounds $\bar{g}$ and evaluation points $\bar{\phi}$ of the fluctuation fields:

相应地，对于所有背景 $\bar{g}$ 和涨落场的评估点 $\bar{\phi}$ ，涨落关联函数 $\Gamma^{(0,n)}\left[\bar{g}\bar{\phi}\right]$ 构成一个封闭的流系统:

$$\partial_t \Gamma_k^{(0,m)}\left[\bar{g}, \bar{\phi}\right] = \text{Flow}^{(0,m)}\left[\bar{g}, \left\{\Gamma_k^{(0,j)}\left[\bar{g}, \bar{\phi}\right]\right\}\right], \quad \text{with} \ \ 0 \leq j \leq m+2.$$

(44)

This leaves us with the following hierarchy: the only closed systems of flow equations are that of the fluctuation correlation functions $\Gamma^{(0,m)}$, and the computation of any other correlation functions requires this set as input. In short, the computation of the fluctuation correlation functions is at the core of any computation in the fRG approach to gauge-fixed metric quantum gravity.

由此我们得到下述层级结构: 只有涨落关联函数 $\Gamma^{(0,m)}$ 的流方程构成封闭系统，计算任何其他关联函数都需要将这套系统作为输入。简而言之，在规范固定度规量子引力的 fRG 方法中，涨落关联函数的计算是所有计算的核心。

All these properties are also visible in the example in Fig. 2: The flow of the graviton two-point function contains a tadpole diagram containing the graviton four-point function. In turn, the flow of the graviton four-point function contains a tadpole diagram containing the graviton six-point function. This exemplifies the hierarchy given in (44). By taking derivatives with respect to the background field, we only add external background-field legs to the diagrams but never any internal background propagator and therefore the hierarchy in (43) holds.

所有这些性质也都可以在图 2 的示例中看出: 引力子两点函数的流包含一个含引力子四点函数的蝌蚪图，而引力子四点函数的流又包含一个含引力子六点函数的蝌蚪图，这就是 (44) 中给出层级结构的示例。通过对背景场求导，我们仅给图添加外背景场线，不会添加任何内背景传播子，因此 (43) 中的层级结构成立。

In a final step, the flows (41) of the correlation functions have to be projected on the flow of the dressing functions $\lambda^{(i)}_{n,m}$ in (38) in a given expansion of the effective action. The projection is done by contracting the flows (41) with all the tensors $\mathcal{T}^{(i)}_{(n,m)}$ in a given tensor basis. In most cases, such a basis is not diagonal: the basis elements overlap. Then, a further diagonalization step is required. For the $\lambda^{(i)}_{0,m}$, which is the expansion used in the fluctuation approach, this leads to the coupled flows:

最后一步中，必须在给定的有效作用量展开下，将关联函数的流 (41) 投影到 (38) 中修饰函数 $\lambda^{(i)}_{n,m}$ 的流上。投影通过将流 (41) 与给定张量基中的所有张量 $\mathcal{T}^{(i)}_{(n,m)}$ 缩并完成。多数情况下，这类基不是对角化的: 基元之间存在重叠，因此需要额外的对角化步骤。对于涨落方法中使用的展开 $\lambda^{(i)}_{0,m}$ ，由此得到耦合流如下:

$$\partial_t \lambda^{(i)}_{0,m} = \mathrm{Flow}_{\lambda^{(i)}_{0,m}}\left[\{\lambda_{0,j}\}\right], \text{ with } 0 \leq j \leq m+2, \tag{45}$$

where we have used the hierarchy (44). Evidently, (45) constitutes a closed set of flow equations. It is at the core of the flow equations of the mixed couplings including the dressings, $\lambda^{(i)}_{n,0}$, of the background effective action (17). With (43), we are led to

其中我们用到了层级关系 (44)。显然，(45) 构成了封闭的流方程组。它是包含背景有效作用量 (17) 的修饰 $\lambda^{(i)}_{n,0}$ 在内的混合耦合流方程的核心。结合 (43)，我们得到

$$\partial_t \lambda^{(i)}_{n,m} = \mathrm{Flow}_{\lambda^{(i)}_{n,m}}\left[\{\lambda_{l,j}\}\right], \text{ with } 0 \leq l \leq n \text{ and } 2 \leq j \leq m+2. \tag{46}$$

Trivially, hierarchies (45) and (46) of the flows of the dressings $\lambda_{n,m}$ carry the same properties as that of the full correlation functions: the system of flows for fluctuation dressings $\lambda_{0,m}$ is closed and is the core of the flows of the mixed dressings. We also remark that the steps for the computation of the flows (45) and (46) can be implemented with the respective computer algebra packages for deriving the flows as well as contracting them: DoFun [100,101], QMeS [102], FormTracer [103,104], and VertExpand [105].

显然，修饰 $\lambda_{n,m}$ 的流层级 (45) 和 (46) 与全关联函数的流层级具有相同性质：涨落修饰 $\lambda_{0,m}$ 的流系统是封闭的，是混合修饰流的核心。我们还要指出，流 (45) 和 (46) 的计算步骤可以通过对应的计算机代数包实现，这些包可用于推导流并对其缩并:DoFun [100,101]、QMeS [102]、FormTracer [103,104] 和 VertExpand [105]。

The momentum-dependent dressings $\lambda_{n,m}^{(i)}(\mathbf{p}_{n+m})$ contain the full information of the quantum field theory. It is nonetheless useful to study their relation to curvature invariants. For example, the tensor structure of the transverse-traceless graviton two-point function, $\mathcal{T}_{0,h_{tt}h_{tt}}$, only has overlap with three tensor structures in an expansion about the flat background:

依赖动量的修饰 $\lambda_{n,m}^{(i)}(\mathbf{p}_{n+m})$ 包含了量子场论的全部信息。研究它们与曲率不变量的关系仍有重要意义。例如，横迹零引力子两点函数的张量结构 $\mathcal{T}_{0,h_{tt}h_{tt}}$，在平坦背景展开中仅与三个张量结构存在重叠:

$$\mathcal{T}_{0,h_{tt}h_{tt}} \sim \sqrt{\pm g}, \sqrt{\pm g}R, \sqrt{\pm g}C_{\mu\nu\rho\sigma} f_C\left(\nabla^2\right) C^{\mu\nu\rho\sigma}. \tag{47}$$

Similarly, the tensor structure of the scalar graviton two-point function, $\mathcal{T}_{0,h_s h_s}$, also only has overlap with three tensor structures:

类似地，标量引力子两点函数的张量结构 $\mathcal{T}_{0,h_s h_s}$ 也仅与三个张量结构存在重叠:

$$\mathcal{T}_{0,h_s h_s} \sim \sqrt{\pm g}, \sqrt{\pm g}R, \sqrt{\pm g}R f_R\left(\nabla^2\right) R. \tag{48}$$

The functions f_R and f_C have been called form factors [16,106-110]. These functions can be computed from solutions of the dressings $\lambda_{0,h_{tt}h_{tt}}$ and $\lambda_{0,h_s h_s}$, which has been done at the fixed point [111-115] and recently also at vanishing cutoff scales [82,92] in Euclidean and Lorentzian signature, which will be discussed in detail in sections "Momentum-Dependent Correlation Functions" and "Lorentzian Quantum Gravity." Many expansion schemes focus on the transverse-traceless graviton mode, which is typically the dominant contribution. In this expansion, the tensor structure $\sqrt{\pm g}R f_R\left(\nabla^2\right) R$ can only be the accessed order h_{tt}^4. This was used in [114] to show that R^2 fluctuations are relevant, while C^2 fluctuations are irrelevant.

函数 f_R 和 f_C 被称为形状因子 [16,106-110]。这些函数可由修饰 $\lambda_{0,h_{tt}h_{tt}}$ 和 $\lambda_{0,h_s h_s}$ 的解计算得到，相关计算已在不动点处完成 [111-115]，最近也在欧几里得与洛伦兹号差下的零截断标度处完成了计算，我们会在“依赖动量的关联函数”和“洛伦兹量子引力”两节中详细讨论。许多展开方案都聚焦于横迹零引力子模式，它通常是主导贡献。在这类展开中，张量结构 $\sqrt{\pm g}R f_R\left(\nabla^2\right) R$ 只能达到 h_{tt}^4 阶。文献 [114] 利用这一点证明了 R^2 涨落是相关的，而 C^2 涨落是不相关的。

One tensor structure only has overlap with finitely many curvature invariants in an expansion about the flat background. Together with the momentum dependence, this allows disentangling the running of different curvature operators, for example, the running of the universal one-loop beta functions of R^2, C^2, and E [116, 117]. In the other direction, any curvature invariant has overlap with infinitely many $\mathcal{T}_{n,m}^{(i)}$.

This creates a redundancy since they are related by diffeomorphism invariance, or more precisely by BRST invariance. How this redundancy is resolved is discussed in the next section.

在平坦背景展开中，一个张量结构仅与有限个曲率不变量存在重叠。结合动量依赖性质，我们可以分离不同曲率算符的跑动，例如 R^2, C^2 和 E [116, 117] 的普适单圈 beta 函数的跑动。另一方面，任意曲率不变量都与无穷多个 $\mathcal{T}^{(i)}_{n,m}$ 存在重叠。由于这些量由微分同胚不变性 (更准确地说是 BRST 不变性) 相互关联，这就产生了冗余性。下一节将讨论如何解决这种冗余性。

Background Independence and Symmetry Identities

背景独立性与对称恒等式

With the flow equations for the dressings, (45) and (46), we can compute the correlation functions within a given approximation to the full hierarchy of flows. However, the correlation functions or rather some of the dressings $\lambda^{(i)}_{n,m}$ are related by diffeomorphism symmetry constraints, and hence the set of all dressings carries diffeomorphism redundancies. These constraints provide nontrivial relations between background $\Gamma_k^{(n,0)}$ and fluctuation correlation functions $\Gamma_k^{(0,m)}$. These relations guarantee the background independence as well as the diffeomorphism invariance of the theory. In this section, we briefly review the symmetry identities that constrain the set of scale-dependent correlation functions $\left\{\Gamma_k^{(n,m)}\right\}$. We shortly describe the origin and provide the final form of the symmetry identities. For a more detailed discussion of symmetry identities, we refer to [31].

利用修饰场的流方程 (45) 和 (46)，我们可以在全流层级的给定近似下计算关联函数。然而，关联函数——更准确地说，部分修饰场 $\lambda^{(i)}_{n,m}$ ——由微分同胚对称性约束关联，因此所有修饰场的集合自带微分同胚冗余性。这些约束给出了背景 $\Gamma_k^{(n,0)}$ 与涨落关联函数 $\Gamma_k^{(0,m)}$ 之间的非平凡关系。这些关系保证了理论的背景独立性和微分同胚不变性。在本节中，我们简要综述约束标度依赖关联函数集合 $\left\{\Gamma_k^{(n,m)}\right\}$ 的对称恒等式。我们会简述其起源并给出对称恒等式的最终形式。关于对称恒等式的更详细讨论，参见文献 [31]。

The quantum field theory approach to metric quantum gravity is based on the underlying symmetry of diffeomorphism invariance. With a linear metric split, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, see (7); this invariance is encoded in the quantum diffeomorphism transformation:

度规量子引力的量子场论方法建立在微分同胚不变性这一基本对称性之上。在线性度规分解 $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ (见 (7) 式) 下，该不变性被编码在量子微分同胚变换中:

$$h_{\mu\nu} \to h_{\mu\nu} + \mathcal{L}_\omega \left(\bar{g}_{\mu\nu} + h_{\mu\nu}\right), \ \bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu}. \tag{49}$$

Here $\mathcal{L}_\omega$ is the Lie derivative with respect to some vector field ω_μ , which reads for a rank-two tensor:

此处 $\mathcal{L}_\omega$ 是对应任意向量场 ω_μ 的李导数，对于二阶张量其形式为:

$$\mathcal{L}_\omega T_{\mu\nu} = \omega_\rho \overline{\nabla}^\rho T_{\mu\nu} + T_{\mu\rho} \overline{\nabla}^\rho \omega_\nu + T_{\nu\rho} \overline{\nabla}^\rho \omega_\mu, \tag{50}$$

with the background covariant derivative $\bar{\nabla}$. The presence of the background metric triggers an auxiliary symmetry, the background diffeomorphism invariance given by the transformation:

其中 $\bar{\nabla}$ 是背景协变导数。背景度规的存在引出了一个辅助对称性，即由下述变换给出的背景微分同胚不变性:

$$h_{\mu\upsilon} \to h_{\mu\upsilon} + \mathcal{L}_\omega h_{\mu\upsilon},\ \bar{g}_{\mu\upsilon} \to \bar{g}_{\mu\upsilon} + \mathcal{L}_\omega \bar{g}_{\mu\upsilon}. \tag{51}$$

Both transformations, (49) and (51), imply the same transformation for the full metric $g_{\mu\nu}$ as expected from a diffeomorphism transformation. While (51) is at first only an auxiliary symmetry, it is still related to the dynamical quantum symmetry via the split symmetry of the metric split. The split symmetry includes all transformations that leave the full metric invariant and is given by

正如微分同胚变换所预期的，变换 (49) 和 (51) 对全度规 $g_{\mu\nu}$ 给出了相同的变换。尽管 (51) 最初只是一个辅助对称性，它仍通过度规分解的分解对称性与动力学量子对称性相关联。分解对称性包含所有保持全度规不变的变换，其形式为

$$g(\bar{g}, h) \to g(\bar{g} + \delta\bar{g}, h + \delta h) = g(\bar{g}, h). \tag{52}$$

For example, in the linear split (7), we have $\delta\bar{g} = -\delta h$. This symmetry encodes the background independence of the theory.

例如，在线性分解 (7) 中，我们得到 $\delta\bar{g} = -\delta h$ 。该对称性编码了理论的背景独立性。

Background Independence

背景独立性

Without the regulator function, the background independence of the theory is encoded in the Nielsen identity (NI) [118, 119]:

在没有调节函数的情况下，该理论的背景独立性由尼尔森恒等式 (NI) 编码 [118, 119]:

$$\mathrm{NI} \equiv \frac{\delta\Gamma}{\delta\bar{g}_{\mu\upsilon}} - \frac{\delta\Gamma}{\delta h_{\mu\upsilon}} - \left\langle \left[\frac{\delta}{\delta\bar{g}_{\mu\upsilon}} - \frac{\delta}{\delta\hat{h}_{\mu\upsilon}} \right] \left(S_{\mathrm{gf}} + S_{\mathrm{gh}} \right) \right\rangle = 0, \tag{53}$$

where $h_{\mu\upsilon} = \langle \hat{h}_{\mu\upsilon} \rangle$. For this identity, we have used the linear split. This identity has to hold at vanishing cutoff scales after the integrating out of all quantum fluctuations. On the quantum equations of motions, the last term in (53) vanishes, which implies that a solution to the quantum equation of motion is also a solution to the background equation of motion, $\delta\Gamma/\delta h = \delta\Gamma/\delta\bar{g}$.

其中 $h_{\mu\upsilon} = \langle \hat{h}_{\mu\upsilon} \rangle$ 。我们推导该恒等式时采用了线性拆分。积分掉所有量子涨落后，该恒等式必须在截断尺度趋近于零时成立。在量子运动方程下，式 (53) 的最后一项为零，这意味着量子运动方程的解同时也是背景运动方程的解，$\delta\Gamma/\delta h = \delta\Gamma/\delta\bar{g}$ 。

At finite cutoff scales, the NIs are modified by the presence of the regulator, leading to the modified NIs (mNIs):

在有限截断尺度下，尼尔森恒等式会因调节项的存在发生修正，得到修正后的尼尔森恒等式 (mNI):

$$\mathrm{mNI}_k \equiv \mathrm{NI}_k - \frac{1}{2}\,\mathrm{Tr}\,\frac{\delta\sqrt{\bar{g}}R_k}{\sqrt{\bar{g}}\delta\bar{g}_{\mu\nu}}G_k = 0. \tag{54}$$

In terms of flowing correlation functions $\Gamma^{(n,m)}$ and the corresponding dressing functions of the tensor structures $\lambda^{(i)}_{n,m}$, see (46); the mNI relates $\Gamma^{(n+1,m)}$ with $\Gamma^{(n,m+1)}$ where the last fluctuation derivative is with respect to the fluctuation metric h . For example, the background propagator $\Gamma^{(2,0)}$ can be determined with the knowledge of the fluctuation propagator $\Gamma^{(0,hh)}$ and vice versa; see [120] for first results in that direction. Note however that the relation is nontrivial due to the expectation value in (53) and the trace in (54). The background propagator can also be computed directly from the flow for a given fluctuation propagator but not the other way around due to the dynamical nature of the fluctuation field and the auxiliary nature of the background metric; see (43). mNIs have been discussed in detail in gravity, gauge theories, and scalar theories; see [3, 15, 24, 69, 77, 78, 120-137].

用流关联函数 $\Gamma^{(n,m)}$ 和张量结构对应的修饰函数 $\lambda^{(i)}_{n,m}$ 表示 (见式 (46))，修正后的尼尔森恒等式关联了 $\Gamma^{(n+1,m)}$ 与 $\Gamma^{(n,m+1)}$ ，其中最后一项涨落导数是对涨落度规 h 求得的。例如，已知涨落传播子 $\Gamma^{(0,hh)}$ 即可确定背景传播子 $\Gamma^{(2,0)}$ ，反之亦然；相关早期结果参见文献 [120]。但请注意，由于式 (53) 中的期望值和式 (54) 中的迹，该关系是非平凡的。给定涨落传播子后，背景传播子也可以直接从流方程计算得到，但反过来却不行，这是因为涨落场是动力学量，而背景度规是辅助量；参见式 (43)。修正后的尼尔森恒等式已经在引力、规范理论和标量理论中得到了详细讨论；参见文献 [3, 15, 24, 69, 77, 78, 120-137]。

Diffeomorphism Invariance

微分同胚不变性

The auxiliary background diffeomorphism invariance (51) remains unbroken by gauge-fixing and regularization. The physical quantum diffeomorphism invariance (49) turns into a BRST symmetry due to the gauge fixing, which is then further modified by the regulator. The related symmetry identities are called (modified) Slavnov-Taylor identities ((m)STI) [138, 139]. They encode physical diffeomorphism invariance.

辅助背景微分同胚不变性 (51) 不会被规范固定和正则化破坏。物理量子微分同胚不变性 (49) 因规范固定转变为 BRST 对称性，随后又被调节子进一步修改。相关的对称恒等式称为 (修正的) 斯拉夫诺夫-泰勒恒等式 ((m)STI) [138, 139]，它们编码了物理微分同胚不变性。

In case of the linear gauge-fixing condition (9), the generator of BRST transformation (or BRST operator) denoted by $\mathfrak{s}$, including the Nakanishi-Lautrup field λ_μ , is given by

对于线性规范固定条件 (9)，包含中西-劳特鲁普场 λ_μ 的 BRST 变换生成元 (或称 BRST 算符) 记为 $\mathfrak{s}$ ，形式如下:

$$\mathfrak{s}\left(\bar{g}_{\mu\nu}, h_{\mu\nu}, c_\mu, \bar{c}_\mu, \lambda_\mu, \phi_{\text{mat}}\right) = \left(0, \mathcal{L}_c\left(\bar{g}_{\mu\nu} + h_{\mu\nu}\right), c_\rho \overline{\nabla}^\rho c_\mu, \lambda_\mu, 0, \mathfrak{s}\phi_{\text{mat}}\right).$$

(55)

Here, the vector field ω_μ in the Lie derivative (50) is given by the ghost field, $\omega_\mu = \ c_\mu$; for more details on the setup and the condensed notation used below, see [24]. The Nakanishi-Lautrup field λ_μ transforms trivially under the BRST transformation, $\mathfrak{s}\lambda_\mu = 0$. The classical gauge-fixed action including the gauge-fixing and the ghost action is invariant under this transformation, $\mathfrak{s}\left(S_{\text{grav}} + S_{\text{gf}} + S_{\text{gh}}\right) = 0$. Furthermore, $\mathfrak{s}$ is a nilpotent operator with $\mathfrak{s}^2 = 0$.

此处，李导数 (50) 中的矢量场 ω_μ 由鬼场 $\omega_\mu = \ c_\mu$ 给出；关于本文的设定以及下文所用的缩记符号，详见 [24]。中西-劳特鲁普场 λ_μ 在 BRST 变换下平凡变换，即 $\mathfrak{s}\lambda_\mu = 0$ 。包含规范固定项与鬼作用量的经典规范固定作用量在该变换下保持不变，即 $\mathfrak{s}\left(S_{\text{grav}} + S_{\text{gf}} + S_{\text{gh}}\right) = 0$ 。此外，$\mathfrak{s}$ 是幂零算符，满足 $\mathfrak{s}^2 = 0$ 。

For the derivation of the STI, we include a source term $Q^a\mathfrak{s}\widehat{\phi}_a$ for the BRST variations of the fields in the generating functional. Expressed in terms of the effective action (see [24]), the STI in the absence of the cutoff term reads

为推导 STI，我们在生成泛函中为场的 BRST 变分引入源项 $Q^a\mathfrak{s}\widehat{\phi}_a$ 。用有效作用量表示 (参见 [24])，无截断项时的 STI 形式为:

$$\text{STI} \equiv \int \mathrm{d}^4x\sqrt{\bar{g}}\,\frac{\delta\Gamma}{\delta\phi_a}\frac{\delta\Gamma}{\delta Q^a} = 0. \tag{56}$$

This equation is known as the quantum-master equation. The BRST variation of the effective action is given by $\delta\Gamma/\delta Q^a = \left\langle\mathfrak{s}\widehat{\phi}_a\right\rangle$. These variations can be interpreted as generalized vertices of the theory.

该方程被称为量子主方程。有效作用量的 BRST 变分由 $\delta\Gamma/\delta Q^a = \left\langle\mathfrak{s}\widehat{\phi}_a\right\rangle$ 给出。这些变分可以解释为该理论的广义顶点。

Equation (56) encodes diffeomorphism invariance at $k = 0$ where the regulator vanishes. At finite cutoff scales, an additional regulator contribution has to be taken into account, and we are led to the mSTI:

方程 (56) 编码了调节子趋于零时 $k = 0$ 处的微分同胚不变性。在有限截断标度下，必须额外考虑调节子贡献，于是我们得到 mSTI:

$$\text{mSTI} \equiv \text{STI} - \operatorname{Tr} R_k \frac{\delta^2\Gamma_k}{\delta Q\delta\phi} G_k = 0. \tag{57}$$

In terms of flowing correlation functions $\Gamma^{(n,m)}$, the mSTI relates different fluctuation correlation functions with each other. Some of the properties of the mSTI are theory-dependent, but most of them are generic: mSTIs in the presence and absence of background fields in gravity and gauge theories have been discussed in detail in $[15, 24, 25, 77, 78, 102, 121-123, 130, 140-159]$.

用流动关联函数 $\Gamma^{(n,m)}$ 表示，mSTI 将不同涨落关联函数联系起来。mSTI 的部分性质依赖于具体理论，但大部分性质是通用的: 引力和规范理论中带背景场与不带背景场情况下的 mSTI 已在 [15, 24, 25, 77, 78, 102, 121 − 123, 130, 140 − 159] 中得到详细讨论。

Wrap-Up

总结

The investigations of the last sections leave us with the following structure of correlation functions and flows in functional approaches to metric quantum gravity:

前几节的研究为我们给出了度量量子引力泛函方法中关联函数与流的下述结构:

The required regularization and gauge-fixing introduces inevitably a background metric $\bar{g}_{\mu\nu}$, and hence two sets of fields, the background fields $\bar{\phi}$ and the dynamical fluctuation fields ϕ . The flows of the $\Gamma^{(n,m)}$ can be derived (see (43)), and it is the dynamical fluctuation correlation functions $\Gamma^{(0,m)}$ that drive all of these flows. In addition to the flow equations, we have symmetry relations between the correlation functions. The mNI (see (54)) relates background correlation function $\Gamma^{(n,0)}$ with fluctuation ones $\Gamma^{(0,m)}$ and ensures background independence. The mSTI (see (57)) relates different fluctuation correlation functions and ensures the BRST symmetry of the diffeomorphism invariance.

所需的正则化与规范固定不可避免地会引入背景度量 $\bar{g}_{\mu\nu}$ ，由此得到两组场: 背景场 $\bar{\phi}$ 和动力学涨落场 ϕ 。我们可以推导出 $\Gamma^{(n,m)}$ 的流 (见式 (43))，驱动所有这些流的正是动力学涨落关联函数 $\Gamma^{(0,m)}$ 。除流方程外，关联函数之间还存在对称性关系。修正的尼尔森恒等式 (mNI，见式 (54)) 将背景关联函数 $\Gamma^{(n,0)}$ 与涨落关联函数 $\Gamma^{(0,m)}$ 相关联，保证了背景独立性。修正的斯莱文-泰勒恒等式 (mSTI, 见式 (57)) 将不同的涨落关联函数相关联，保证了微分同胚不变性的 BRST 对称性。

In principle, it would be a great simplification if the dependence on the background fields could be eliminated. However, the nonlinear nature of the diffeomorphism transformation does not allow us to derive the effective action in a separable form such as

原则上，如果能消除对背景场的依赖，问题就会得到极大简化。然而，微分同胚变换的非线性性质导致我们无法将有效作用量整理为如下可分离形式

$$\Gamma\left[\bar{\phi}, \phi\right] \approx \Gamma\left[\bar{\phi} + \phi\right] + S_{\mathrm{gf}}\left[\bar{g}, h\right] + S_{\mathrm{gh}}\left[\bar{g}, h, c\right]. \tag{58}$$

Equation (58) is known as the background-field approximation and is commonly used in computations in the fRG approach to metric quantum gravity. It is readily seen that the flow equation (25) is diffeomorphism-invariant at $h = 0$ and reduces to a flow of $\Gamma\left[\bar{\phi}, 0\right]$. Note, however, that the flow (25) with $\phi \neq 0$ does not sustain the separability given in (58) due to both the gauge-fixing sector $S_{\mathrm{gf}} + S_{\mathrm{gh}}$ and the cutoff term.

式 (58) 就是所谓的背景场近似，它普遍应用于度量量子引力的泛函重整化群 (fRG) 方法计算中。不难看出，流方程 (25) 在 $h = 0$ 处满足微分同胚不变性，可约化为 $\Gamma\left[\bar{\phi}, 0\right]$ 的流。但需要注意，由于规范固定区 $S_{\rm gf} + S_{\rm gh}$ 和截断项的存在，带有 $\phi \neq 0$ 的流 (25) 无法维持 (58) 给出的可分离形式。

Hence, while (58) has the appealing feature of diffeomorphism invariance (up to the classical gauge-fixing term) and leads to a seemingly gauge-invariant flow equation, its diffeomorphism invariance rests on an approximation that explicitly breaks diffeomorphism invariance. This is readily seen by inserting (58) into the STIs. This leads us to the seemingly self-contradictory situation that the diffeomorphism invariance of (58) signals is breaking. Indeed, the fluctuation correlation functions are required to be not diffeomorphism-covariant for $\Gamma\left[\bar{\phi}\right]$ carrying physical diffeomorphism invariance [160].

因此，尽管 (58) 具有微分同胚不变性 (直到经典规范固定项) 这一吸引人的特点，还能得到看似规范不变的流方程，但其微分同胚不变性建立在一个显式破缺微分同胚不变性的近似之上。将 (58) 代入斯莱文-泰勒恒等式 (STI) 即可看出这一点。这就将我们带入了一个看似自相矛盾的情形:(58) 的微分同胚不变性本身就预示着它的破缺。事实上，对于承载物理微分同胚不变性的 $\Gamma\left[\bar{\phi}\right]$，要求涨落关联函数不具有微分同胚协变性 [160]。

There have been many attempts over the past decades to enforce diffeomorphism covariance, or more generally gauge invariance of general gauge theories, of fluctuation correlation functions in a gauge-fixed approach via nonlinear transformations. The latter are either directly applied or indirectly implemented with diffeomorphism constraints. For a (an incomplete) list of respective references within the fRG approach, see [76-78, 80, 157, 161-163]. Loosely speaking the price to pay for diffeomorphism covariance of fluctuation correlation functions in a gauge-fixed approach is non-locality. We emphasize in this context that while commonly dropped in the discussions of general gauge theory, it is gauge invariance together with locality, which is required; for a more detailed discussion, see [30]. In summary, while there is remarkable progress, it is fair to say that the diffeomorphism-covariant functional approaches are still in their infancy concerning the evaluation of their overall consistency and structure.

过去几十年中，已有诸多研究尝试通过非线性变换，在规范固定方法中强制要求涨落关联函数满足微分同胚协变性, 更一般地说, 满足广义规范理论的规范不变性。这类非线性变换要么直接应用, 要么通过微分同胚约束间接实现。关于 fRG 框架下相关参考文献 (不完全列表)， 参见 [76-78, 80, 157, 161-163]。宽泛地说，在规范固定方法中获得涨落关联函数的微分同胚协变性，代价就是非定域性。我们在此需要强调，尽管在广义规范理论的讨论中这一点常常被忽略，我们实际要求的是同时满足规范不变性和定域性，更详细的讨论参见 [30]。总而言之，尽管目前已经取得了显著进展，公允地说，从整体一致性和结构构建的层面来看，满足微分同胚协变的泛函方法仍处于发展初期。

The fluctuation approach discussed in this contribution aims at solving the closed set of flow equations for fluctuation correlation functions; see (44). The flows have to be solved with the boundary condition given by the STI at vanishing cutoff scale. The NI can then be used for the computation of the background correlation functions. This is an ambitious goal but it can be achieved for given orders in a systematic expansion scheme that does not spoil diffeomorphism invariance from the outset.

本文讨论的涨落方法旨在求解涨落关联函数的封闭流方程组，参见式 (44)。这些流需要在截断标度为零时，以斯莱文-泰勒恒等式给出的边界条件进行求解。随后可以利用尼尔森恒等式计算背景关联函数。这是一个宏大的目标，但可以在系统展开方案的给定阶下实现，且该方案从一开始就不会破坏微分同胚不变性。

Fluctuation Approach

涨落方法

In sections "Functional Renormalization Group Approach to Metric Quantum Gravity" and "Background Independence and Symmetry Identities," we have discussed the general fRG approach to asymptotically safe metric quantum gravity. The goal is to compute the effective action $\Gamma[\bar{g},\phi]$,(14), and the derived diffeomorphism-invariant background effective action $\bar{\Gamma}[g,\phi_{\text{mat}}]$,(17). This task is tantamount to that of the respective correlation functions $\Gamma^{(n,m)}$,(27), whose flows are governed by (25). As discussed in sections "Functional Renormalization Group Approach to Metric Quantum Gravity" and "Background Independence and Symmetry Identities," both tasks require the computation of the fluctuation correlation functions in a given background at the chosen expansion point $\phi=\bar{\phi}$. In general, this reads

在“度规量子引力的泛函重整化群方法”与“背景无关性与对称性恒等式”两节中，我们已经讨论了渐近安全度规量子引力的一般 fRG 方法。我们的目标是计算有效作用量 $\Gamma[\bar{g},\phi]$，见 (14)，以及推导出的微分同胚不变的背景有效作用量 $\bar{\Gamma}[g,\phi_{\text{mat}}]$， 见 (17)。该任务等价于求解对应关联函数 $\Gamma^{(n,m)}$ ，见 (27)，这些关联函数的流由 (25) 支配。正如在“度规量子引力的泛函重整化群方法”与“背景无关性与对称性恒等式”两节中所讨论的，两项任务都要求计算选定展开点 $\phi=\bar{\phi}$ 给定背景下的涨落关联函数。一般来说，其形式为

$$\Gamma^{\phi_{a_1}\cdots\phi_{a_n}}\left[\bar{g},\bar{\phi}\right](\mathbf{x}_n)=\left.\frac{\delta^n\Gamma[\bar{g},\phi]}{\delta\phi_{a_1}(x_1)\cdots\delta\phi_{a_n}(x_n)}\right|_{\phi=\bar{\phi}}. \tag{59}$$

The results for (59) are the expansion coefficients of the effective action in an expansion about a generic expansion point $\bar{\phi}$; for $\bar{\phi}=0$ see (37). For $\bar{\phi}\neq 0$, we write schematically

(59) 的结果就是有效作用量在关于一般展开点 $\bar{\phi}$ 展开下的展开系数；关于 $\bar{\phi}=0$ 参见 (37)。对于 $\bar{\phi}\neq 0$，我们概要写出

$$\Gamma[\bar{g},\phi]=\sum_{n=0}^{\infty}\int_{\mathbf{x}_n}\Gamma^{\phi_{a_1}\cdots\phi_{a_n}}\left[\bar{g},\bar{\phi}\right](\mathbf{x}_n)\left[\left(\phi-\bar{\phi}\right)_{a_1}(x_1)\cdots\left(\phi-\bar{\phi}\right)_{a_n}(x_n)\right],$$

(60)

The fluctuation correlation functions (59) are then used to compute the complete system of correlation functions $\Gamma^{(n,m)}$, which is required for determining the background effective action $\bar{\Gamma}[g,\phi_{\text{mat}}]$.

涨落关联函数 (59) 随后被用于计算关联函数 $\Gamma^{(n,m)}$ 的完整体系,这是确定背景有效作用量 $\bar{\Gamma}[g,\phi_{\text{mat}}]$ 所必需的。

Expansion Schemes and RG-Invariant Vertex Dressings

展开方案与 RG 不变顶点修饰

The computation of the whole system of correlation functions without relying on the background approximation has been baptized fluctuation approach for the reason that the core of any systematic computation is the fluctuation correlation functions (59). Evidently, one needs a systematic approximation scheme for any computation, but the fluctuation approach only rests on such a scheme and no further approximation.

不依赖背景近似计算关联函数完整体系的方法被命名为涨落方法, 原因是任何系统化计算的核心都是涨落关联函数 (59)。显然，任何计算都需要系统化近似方案，但涨落方法仅依赖这类方案，无需额外近似。

Equation (59) also comprises the background correlation functions and the vacuum correlation function $\Gamma^{(0,0)}\left[\bar{g},\overline{\phi}\right]$ at $h, c, \bar{c} = 0$, which is nothing but the background effective action (17). However, this is more a conceptual remark rather than the one with computational significance, as already the computation of fluctuation correlation functions in symmetric background metrics is a very challenging task. Therefore, in most computations, we resort to a double expansion scheme both in terms of fluctuation fields about a simple background $\overline{\phi}$ and an expansion about a simple background metric $\bar{g}_{\mu\nu}$. We emphasize in this context that any expansion of the flow equation is only a tool for computing correlation functions; the expansion point itself is not required to have any physical significance such as the solution of the quantum equations of motion $g_{\text{EoM}}, \phi_{\text{EoM}}$ with

方程 (59) 还包含背景关联函数以及 $h, c, \bar{c} = 0$ 处的真空关联函数 $\Gamma^{(0,0)}\left[\bar{g},\overline{\phi}\right]$，后者正是背景有效作用量 (17)。不过这更多是概念层面的说明，不具备计算层面的意义，因为即便仅计算对称背景度规下的涨落关联函数，也是一项极具挑战性的工作。因此，在大多数计算中我们采用双展开方案: 既围绕简单背景 $\overline{\phi}$ 对涨落场展开, 又围绕简单背景度规 $\bar{g}_{\mu\nu}$ 展开。在此我们需要强调, 流方程的任何展开都只是计算关联函数的工具；展开点本身不需要具备任何物理意义，比如量子运动方程 $g_{\text{EoM}}, \phi_{\text{EoM}}$ 的解

$$\frac{\delta \Gamma}{\delta \bar{g}} = 0 = \frac{\delta \Gamma}{\delta \phi}, \quad \text{for} \quad (\bar{g}, \phi) = (g_{\text{EoM}}, \phi_{\text{EoM}}) \,. \tag{61}$$

Still, the further away the expansion point is from the solution to the equations of motion, the more the stability of the expansion scheme is at stake.

但展开点离运动方程的解越远，展开方案的稳定性受影响就越大。

In summary, the fluctuation approach is the general fRG approach to metric quantum gravity without resorting to further approximations. In explicit computations, we have to specify the expansion scheme chosen: in almost all computational applications to date, the background metric has been set to the flat metric, either Euclidean or Minkowski. We emphaisze that such a choice is related to a curvature expansion in the background effective action, the latter also being an expansion about the flat background. However, in the fluctuation approach with $\bar{g} = \eta$, typically momentum dependences are computed that give access to covariant momentum dependences in the background effective action.

综上，涨落方法是不依赖额外近似的度量量子引力泛函重整化群 (fRG) 通用方法。在具体计算中，我们必须明确选定的展开方案: 迄今为止几乎所有计算应用中，都将背景度规设为平坦度规，包括欧几里得平坦度规和闵可夫斯基平坦度规。我们强调，该选择对应背景有效作用量中的曲率展开，而曲率展开本身就是围绕平坦背景的展开。但在引入 $\bar{g} = \eta$ 的涨落方法中，通常会计算动量依赖项，这些项可用于得到背景有效作用量中的协变动量依赖关系。

The above discussion specifies the background and hence the expansion in (60) used for most computations, but it does not specify the split between the background metric $\bar{g}$ and the fluctuation field h . While the approach works with any split, in this contribution, we focus on the linear split (7) with a nontrivial prefactor. A very convenient choice for the linear metric split is given by

上述讨论明确了大多数计算所用的背景，也即 (60) 中的展开，但并未明确背景度规 $\bar{g}$ 和涨落场 h 之间的拆分方式。虽然该方法对任意拆分都适用，在本文中我们聚焦于带非平凡前置因子的线性拆分 (7)。线性度规拆分的一个非常方便的选择为

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{Z_h G_\mathrm{N}} h_{\mu\nu}, \tag{62}$$

with the wave function Z_h and the Newton coupling G_N . While the split (62) works generically, it is natural for asymptotic safety based on the IR Einstein-Hilbert action (2). This is already indicated by the occurrence of the Newton coupling leading to dimension 1 fluctuation fields $h_{\mu\nu}$. While it could be substituted by any dimension -2 coupling, the choice (62) removes the G_N prefactors from $S^{(2)}_\mathrm{EH}$. Note, in this context, that specifying the classical action is tantamount to setting the classically propagating degrees of freedom: in the classical Einstein-Hilberg action, (2), this is only the massless graviton, while in the classical Stelle action, (6), this also includes the massive scalar mode and the massive spin-two ghost. We emphasize that while it is important to set the initial propagating degrees of freedom, the quantum degrees of freedom can be different. Furthermore, using the Einstein-Hilbert action in the path integral is not the same as working in the Einstein-Hilbert truncation. In the latter, only terms related to the cosmological constant and the Ricci scalar are taken into account, while here all generated contributions are considered. Importantly, there are finite higher-curvature contributions in the limit $k \to \infty$: the difference is that these contributions do not have an ultra-local part; see the next section, section "Apparent Convergence," for more details.

其中波函数为 Z_h ，牛顿耦合为 G_N 。拆分 (62) 具有普适性，同时对于基于红外爱因斯坦-希尔伯特作用量 (2) 的渐近安全而言十分自然，这一点从牛顿耦合的引入就能体现——它导出了量纲为 1 的涨落场 $h_{\mu\nu}$ 。尽管该位置也可以替换为任意量纲为-2 的耦合，但选择 (62) 可将 G_N 前置因子从 $S^{(2)}_\mathrm{EH}$ 中消去。需要注意，指定经典作用量等价于确定经典传播自由度: 在经典爱因斯坦-希尔伯特作用量 (2) 中，仅包含无质量引力子，而在经典施泰勒作用量 (6) 中，还包含有质量标量模和有质量自旋二鬼。我们强调，虽然确定初始传播自由度很重要，但量子自由度可以与之不同。此外，在路径积分中使用爱因斯坦-希尔伯特作用量不等价于采用爱因斯坦-希尔伯特截断: 后者仅考虑与宇宙学常数和里奇标量相关的项，而我们此处会包含所有生成的贡献。重要的是，在 $k \to \infty$ 极限下存在有限高曲率贡献；区别在于这些贡献不包含超局域部分，更多细节参见下一节“表观收敛”。

Note that on the quantum level, each mode carries a different wave function and Z_h would be a matrix. For now, we discuss approximations with a uniform wave function Z_h and discuss the general case later. The wave function is dimensionless and depends on the flat background on the momentum $Z_h = Z_h(p)$. The Newton coupling carries the mass dimension $[G_\mathrm{N}] = -2$, and therefore the fluctuation field carries the mass dimension $\left[h_{\mu\nu}\right] = 1$ which is that of a standard scalar field in four dimensions. The rescaling in (62) guarantees a canonical form of the graviton propagator, for example, for the transverse-traceless part

请注意，在量子层面，每个模式对应不同的波函数，因此 Z_h 是一个矩阵。目前我们先讨论均匀波函数 Z_h 近似，一般情况留待后叙。波函数是无量纲的，依赖于动量 $Z_h = Z_h(p)$ 处的平坦背景。牛顿耦合具有质量量纲 $[G_\mathrm{N}] = -2$，因此涨落场的质量量纲为 $\left[h_{\mu\nu}\right] = 1$，与四维空间中标准标量场的量纲一致。式 (62) 的重标度保证了引力子传播子的正则形式，例如对横向无迹部分而言就是如此

$$G_{h_{tt}h_{tt}} \propto \frac{1}{Z_h(p)(p^2+\mu)}, \tag{63}$$

where μ is the graviton mass parameter, which is related to the cosmological constant through a mNI. Equation (63) holds similarly for the other modes of the graviton. We emphasize that these rescalings are used to simplify the relation between fluctuation correlation functions and background correlation functions; they do not affect or change the expansion scheme.

其中 μ 是引力子质量参数，它通过 mNI 与宇宙学常数关联。式 (63) 对引力子的其他模式同样成立。我们需要强调，这些重标度仅用于简化涨落关联函数与背景关联函数之间的关系，不会影响或改变展开方案。

Inserting (62) on both sides of the functional flow (25) leads to a hierarchy of coupled integral-differential equations for fluctuation correlation functions. A diagrammatic depiction of the first five flows for $\Gamma^{(0,0)}, \Gamma^{(0,1)}, \ldots, \Gamma^{(0,4)}$ is given in Fig. 2. This system of equations has been solved in a flat background [114] including the momentum dependence of the correlation functions. It still constitutes the most advanced computation of momentum-dependent fluctuation correlation functions in metric quantum gravity. The fluctuation correlation functions are related to S-matrix elements via their RG-invariant vertex cores $\overline{\Gamma}^{(0,n)}(\mathbf{p}_n)$ with the general definition:

将 (62) 代入泛函流 (25) 两侧，可得到涨落关联函数的耦合积分微分方程层级结构。图 2 给出了 $\Gamma^{(0,0)}, \Gamma^{(0,1)}, \ldots, \Gamma^{(0,4)}$ 前五个流的图示描述。该方程组已在平坦背景下求解 [114]，包含了关联函数的动量依赖。它至今仍是度量量子引力中动量依赖涨落关联函数最先进的计算。涨落关联函数通过其 RG 不变顶点核 $\overline{\Gamma}^{(0,n)}(\mathbf{p}_n)$ 与 S 矩阵元关联，一般定义为:

$$\overline{\Gamma}_k^{(n,\phi_{a_1}\cdots\phi_{a_m})}(\mathbf{p}_{n+m}) := \frac{\Gamma_k^{(n,\phi_{a_1}\cdots\phi_{a_m})}(\mathbf{p}_{n+m})}{\prod_{i=1}^{n} Z_{\bar{g}}^{1/2}(p_i) \prod_{j=1}^{m} Z_{\phi_{a_j}}^{1/2}(p_{n+j})}. \tag{64}$$

Here, the $Z_{\phi_{a_i}}(p_i)$ are the fully momentum-dependent wave functions of the fields ϕ_i with $\Gamma^{(0,2)} \simeq Z(p)\, S^{(0,2)}(p)$; see also (63). The correlation functions (64) can be expanded in terms of the same tensor basis as the $\Gamma^{(n,m)}$ in (38), to wit,

此处，$Z_{\phi_{a_i}}(p_i)$ 是满足 $\Gamma^{(0,2)} \simeq Z(p)\, S^{(0,2)}(p)$ 的场 ϕ_i 的全动量依赖波函数，另见 (63)。关联函数 (64) 可按 (38) 中 $\Gamma^{(n,m)}$ 相同的张量基展开，即

$$\overline{\Gamma}_k^{(n,m)}[\bar{g},0] = \sum_i^{N_{n,m}} \bar{\lambda}_{n,m}^{(i)} \mathcal{T}_{n,m}^{(i)}, \tag{65}$$

with

其中

$$\bar{\lambda}^{(i)}_{n,\phi_{a_1}\cdots\phi_{a_m}}(\mathbf{p}_{n+m}) = \frac{\lambda^{(i)}_{n,\phi_{a_1}\cdots\phi_{a_m}}(\mathbf{p}_{n+m})}{\prod_{i=1}^{n} Z_{\bar{g}}^{1/2}(p_i)\prod_{j=1}^{m} Z_{\phi_{a_j}}^{1/2}(p_{n+j})}. \tag{66}$$

The correlation functions (64) and their dressings (66) are invariant under standard RG transformations that constitute in their most general form reparametrizations of the theory and are typically done via the variation of an RG-scale μ ; for a detailed discussion, see [24]. Such a reparametrizations has to be contrasted with the IR cutoff scaling in the fRG approach, which entails the successive integration of momentum modes. The homogeneous RG equation for the $\bar{\Gamma}^{(n,m)}$ reads

关联函数 (64) 及其修饰 (66) 在标准 RG 变换下保持不变；RG 变换最一般的形式是理论的重参数化，通常通过改变 RG 标度 μ 实现，详细讨论参见文献 [24]。这种重参数化需要与 fRG 方法中的红外截断标度区分开，后者需要逐次积分动量模式。$\bar{\Gamma}^{(n,m)}$ 的齐次 RG 方程为

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\bar{\Gamma}^{(n,m)}(\mathbf{p}_{n+m}) = 0, \;\to\; \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\bar{\lambda}^{(i)}_{n,m}(\mathbf{p}_{n+m}) = 0. \tag{67}$$

The S -matrix is built from the tree-level diagrams with the vertices (64) and the fluctuation propagators. Hence, the scalar dressings $\bar{\lambda}_{0,n}$ of $\bar{\Gamma}^{(0,n)}(\mathbf{p}_n)$ are the nondiagrammatic part of the form factor of the respective S -matrix element. Note, however, that in gauge theories, neither (64) nor its trivial extension to (17) provides the form factors of the S -matrix, but provides RG-invariant momentum-dependent scattering couplings, which are the core objects in the fluctuation approach. Note also that the dressings are called form factors themselves and are at the root of the form factor approach to metric quantum gravity reviewed in [16].

S 矩阵由带顶点 (64) 和涨落传播子的树图构成。因此，$\bar{\Gamma}^{(0,n)}(\mathbf{p}_n)$ 的标量修饰 $\bar{\lambda}_{0,n}$ 是对应 S 矩阵元形状因子的非图部分。但需要注意，在规范理论中，无论是 (64) 还是它对 (17) 的平凡推广，都不直接给出 S 矩阵的形状因子，而是给出 RG 不变的动量依赖散射耦合，这是涨落方法中的核心对象。还需注意，这类修饰本身被称为形状因子，也是文献 [16] 综述的度量量子引力形状因子方法的基础。

It is worth mentioning that the essential RG, developed in [164] and used in asymptotically safe gravity in [165, 166], leads to the definition of rescaled fields that absorb the wave functions and automatically generate the vertices of the type of (65). This also involves the field dependence of the wave functions and other terms and hence includes general field-dependent reparametrizations, the flowing fields. These reparametrizations are included within the general fRG for the effective action derived in [24], based on [167]. For further applications, see [168], where the fRG with flowing fields is used to obtain classical dispersions for the full theory, which optimizes the vertex expansion scheme.

值得一提的是，文献 [164] 中提出、并在 [165, 166] 中应用于渐近安全引力的本质重整化群，给出了吸收波函数的重标度场定义，可自动生成 (65) 形式的顶点。这也涵盖了波函数和其他项的场依赖，因此包含了一般的场相关重参数化，即流动场。这些重参数化已被包含在文献 [24] 基于 [167] 推导出的有效作用量一般泛函重整化群中。更多应用参见 [168]，其中带流动场的泛函重整化群被用于得到完整理论的经典色散，优化了顶点展开方案。

In the remainder of this section, we discuss the stability and convergence of such an expansion, the intricacies of stability investigations and relevant directions at a fixed point for overdetermined systems such as present in gauge theories, and the relations to other approaches.

在本节剩余部分，我们讨论这类展开的稳定性与收敛性，稳定性研究的复杂之处，规范理论这类超定系统在不动点处的相关方向，以及该方法与其他方案的关联。

The convergence of the expansion scheme in terms of correlation functions $\Gamma^{(0,n)}$ hinges on the subdominance of higher-order correlation functions in the flow of lower-order ones. Put differently, one studies the convergence of the results for a low-order correlation function such as the propagator in dependence on the full resolution of successively higher-order correlation functions. This is called apparent convergence. Certainly, it is short of a proof of true convergence which however is absent for any other non-perturbative approach to metric quantum gravity or any other non-perturbative quantum field theory such as QCD for that matter. Note also that even in perturbation theory, apparent convergence is used. However, in the latter case, the expansion at least relies on a small parameter, and we discuss such a parameter in the current expansion scheme in the next section. This discussion is based on an expansion about a flat background, which is the common choice for the background. However, we emphasize that the structural analysis does not depend on this choice. Moreover, in more sophisticated applications, in particular in gravity-matter systems, one typically resorts to a mixed expansion scheme: the effective action is expanded in terms of momentum-dependent correlation functions, but this expansion is augmented by full field-dependent functionals such as the effective potential $V_{\text{eff}}(\phi)$ of some matter field or even the fluctuating graviton; see, e.g., [169-171]. Such approximations have been commonly used specifically in QCD, and the current analysis draws a lot from results obtained there; for a discussion, see [30].

以关联函数 $\Gamma^{(0,n)}$ 展开的方案的收敛性，依赖于低阶关联函数流中高阶关联函数处于次优地位。换句话说，我们在逐次解析更高阶关联函数的基础上，研究传播子这类低阶关联函数结果的收敛性，这被称为表观收敛。可以肯定的是，它并非真收敛的证明，不过目前度量量子引力或量子色动力学这类其他非微扰量子场论的所有非微扰方法都无法给出真收敛证明。另外要注意，即使在微扰论中也会使用表观收敛。但微扰论中的展开至少依赖一个小参数，我们会在下一节讨论当前展开方案中的这类参数。本文讨论基于平直背景展开，这是背景的常用选择，但我们要强调结构分析并不依赖该选择。此外，在更复杂的应用中，尤其是引力-物质系统，通常会采用混合展开方案: 有效作用量按动量相关关联函数展开，同时引入全场相关泛函，比如部分物质场甚至涨落引力子的有效势 $V_{\text{eff}}(\phi)$；参见例如 [169-171]。这类近似在量子色动力学中已被广泛使用，本文的分析也大量借鉴了该领域已有结果，相关讨论参见 [30]。

Apparent Convergence

表观收敛

At the root of convergence or rather apparent convergence of the vertex expansion scheme is the phase space suppression of diagrams with higher-order vertices in the hierarchy of flows in Fig. 2. In a standard renormalizable local quantum field theory with polynomial classical interactions, the vertices can be ordered in terms of a few, typically primitively divergent, vertices $\Gamma_{\text{cl}}^{(0,n)}$ or rather the respective dressings $\lambda_{0,n}^{(\text{i})} = \lambda_{0,n}^{(\text{cl})}$ with a point-like classical interaction core and quantum vertices $\Gamma_{\text{qu}}^{(n)}$ that are only generated by quantum corrections and hence loop diagrams. The dressings of the classical tensor structures can be parametrized as

顶点展开方案的收敛 (更准确地说是表观收敛) 的根源在于，图 2 流层级中高阶顶点图存在相空间压制。在带有多项式经典相互作用的标准可重整化定域量子场论中，顶点可根据少数几个 (通常是原始发散的) 顶点 $\Gamma_{\text{cl}}^{(0,n)}$ 排序，更准确地说，分别是带有点状经典相互作用核的修饰项 $\lambda_{0,n}^{(\text{i})} = \lambda_{0,n}^{(\text{cl})}$，以及仅由量子修正 (即圈图) 产生的量子顶点 $\Gamma_{\text{qu}}^{(n)}$ 。经典张量结构的修饰可以参数化为:

$$\lambda_{0,n}^{(\text{cl})}(\mathbf{x}_n) \propto \lambda_n^{(\text{cl})}\delta(x_1 - x_2)\cdots\delta(x_{n-1} - x_n) + \lambda_{0,n}^{(\text{cl,dis})}(\mathbf{x}_n), \tag{68}$$

where $\lambda^{(\text{cl,dis})}$ carries the part of the dressing without point-like contributions: hence, it is a finite distribution function. Note that its different parts may still contain δ - functions, but not the full product of δ -functions in all positions as in the first term. In momentum space, the first term in (68) gives rise to a constant term in the vertex for $p_i^2 \to \infty$ for all $i = 1, \dots, n$, while $\lambda^{(\text{cl,dis})}$ decays if all momenta are taken to infinity:

其中 $\lambda^{(\text{cl,dis})}$ 承载了不带点状贡献的修饰部分: 因此它是一个有限分布函数。请注意，其不同部分仍可能包含 δ 函数，但不会像第一项那样在所有位置都包含完整的 δ 函数乘积。在动量空间中，式 (68) 的第一项会为所有 $i = 1, \dots, n$ 对应的 $p_i^2 \to \infty$ 顶点产生一个常数项，而当所有动量趋于无穷时 $\lambda^{(\text{cl,dis})}$ 衰减:

$$\lim_{p\to\infty} \lambda_{0,n}^{(\text{cl})}(\mathbf{p}_n)\Big|_{p_i^2=p^2} = (2\pi)^{4n}\lambda_n^{(\text{cl})}\delta(\mathbf{p}_n), \quad \lim_{p\to\infty} \lambda_{0,n}^{(\text{cl,dis})}(\mathbf{p}_n)\Big|_{p_i^2=p^2} = 0, \tag{69}$$

with $\mathbf{p}_n$ as defined in (28). We emphasize that it does not suffice to only take the limit $\mathbf{p}_n^2 \to \infty$: if this limit is accomplished by $p_i^2 \to \infty$ for some of the momenta, while others are kept finite, $\lambda_{0,n}^{(\text{cl,dis})}$ does not necessarily vanish.

其中 $\mathbf{p}_n$ 满足式 (28) 的定义。我们需要强调，仅取极限 $\mathbf{p}_n^2 \to \infty$ 是不够的: 如果该极限仅对部分动量通过 $p_i^2 \to \infty$ 实现，其余动量保持有限，则 $\lambda_{0,n}^{(\text{cl,dis})}$ 不一定会消失。

A prominent non-perturbative example for the above properties is QCD where $\Gamma_{\text{cl}}^{(0,n)}$ with $n = 2, 3, 4$ is given by the inverse propagator and three- and four-point vertex parts that carry the classical tensor structure $S^{(n)}$. The quantum vertices are all other vertices including the parts of the full three- and four-point vertices $\Gamma^{(0,3)}$ and $\Gamma^{(0,4)}$ in (65) with nonclassical tensor structures. We extend this distinction to metric quantum gravity and write in analogy:

上述性质的一个典型非微扰例子是量子色动力学 (QCD)，其中带 $n = 2, 3, 4$ 的 $\Gamma_{\mathrm{cl}}^{(0,n)}$ 由逆传播子，以及承载经典张量结构 $S^{(n)}$ 的三点和四点顶点部分给出。量子顶点是所有其他顶点，包括式 (65) 中完整三点、四点顶点 $\Gamma^{(0,3)}$ 和 $\Gamma^{(0,4)}$ 里带非经典张量结构的部分。我们将该区分推广到度规量子引力，类比写出:

$$\left\{\Gamma^{(0,n)}\left(\mathbf{x}_n\right)\right\} = \left\{\Gamma_{\mathrm{cl}}^{(0,n)}\left(\mathbf{x}_n\right), \Gamma_{\mathrm{qu}}^{(0,n)}\left(\mathbf{x}_n\right)\right\}. \tag{70}$$

In contradistinction to QCD, in metric quantum gravity, each n -point function of gravitons contains a classical part proportional to $S_{\mathrm{EH}}^{(n)}$. However, all these parts are directly related to each other, and the vertex expansion underlying apparent convergence is one about the classical pieces $\Gamma_{\mathrm{cl}}^{(0,n)}\left(\mathbf{x}_n\right)$ of all vertices. While this leaves us seemingly with an infinite set of classical vertices, the dressings of all these vertices are related to each other. In the present example of the Einstein-Hilbert action, the RG-invariant dressings (66) of the fluctuation graviton n -point functions, $\bar{\lambda}_n^{(\mathrm{EH})}$, produce avatars of the Newton coupling $G_n\left(p\right)$:

与 QCD 不同，在度规量子引力中，引力子的每个 n 点函数都包含一个正比于 $S_{\mathrm{EH}}^{(n)}$ 的经典部分。但所有这些部分都直接相互关联，支撑表观收敛的顶点展开是针对所有顶点的经典片段 $\Gamma_{\mathrm{cl}}^{(0,n)}\left(\mathbf{x}_n\right)$ 展开。虽然这看似给我们留下了无穷多的经典顶点，但所有这些顶点的修饰都是相互关联的。在爱因斯坦-希尔伯特作用量的当前例子中，涨落引力子 n 点函数的 RG 不变修饰 $\bar{\lambda}_n^{(\mathrm{EH})}$ 产生了牛顿耦合 $G_n\left(p\right)$ 的化身:

$$\bar{\lambda}_n^{(\mathrm{EH})}\left(p\right) = G_n(p)^{\frac{n-2}{2}} + \bar{\lambda}_n^{(\mathrm{EH,dis})}\left(p\right), \tag{71}$$

where p indicates the evaluation at the symmetric point or another appropriate momentum configuration. Note that the separation on the right-hand side is rather difficult to perform explicitly and is not relevant for any computation: it is only the full vertex dressings that enter any flow. The existence of avatars is directly related to the non-polynomial form of the Einstein-Hilbert action or any other diffeomorphism action and hence is related to diffeomorphism invariance. This complicates the stability analysis, which requires more care in a diffeomorphism- or gauge-invariant theory; see section ”Fixed Point and Critical Exponents.”

其中 p 表示在对称点或其他合适动量构型下的求值。请注意，右侧的拆分很难显式完成，且对任何计算都不相关: 只有完整的顶点修饰才会进入任意流方程。化身的存在直接与爱因斯坦-希尔伯特作用量 (或任意微分同胚作用量) 的非多项式形式相关，因此也与微分同胚不变性相关。这增加了稳定性分析的难度，在微分同胚不变或规范不变理论中需要更谨慎处理；参见“不动点与临界指数”一节。

In summary, this leaves us with a vertex expansion in terms of the quantum corrections of the vertices:

综上，我们最终得到了按顶点量子修正排列的顶点展开:

$$\Gamma_{\mathrm{qu}}^{(0,n)}\left(x_n\right), \tag{72}$$

for all theories. We emphasize that the use of the Einstein-Hilbert action in the above argument was done for the sake of an explicit example. The underlying structure is universal. However, in the vicinity of a nontrivial fixed point such as the UV Reuter fixed point in metric quantum gravity, the ordering in (70) has to be accessed carefully as there we have

适用于所有理论。我们需要强调，上述论证中使用爱因斯坦-希尔伯特作用量只是为了给出具体示例，其底层结构是普适的。但在非平凡不动点 (例如度规量子引力中的 UV 罗伊特不动点) 附近，必须谨慎处理式 (70) 中的排序，因为在该处我们有

$$\Gamma_{\mathrm{cl}}^{(0,n)}(\mathbf{x}_n) \to \Gamma_{\mathrm{fp}}^{(0,n)}(\mathbf{x}_n)\,, \tag{73}$$

where $\Gamma_{\mathrm{fp}}^{(0,n)}$ are the expansion coefficients of the fixed-point action or an approximate fixed-point action with a few relevant terms.

其中 $\Gamma_{\mathrm{fp}}^{(0,n)}$ 是不动点作用量或仅含少数相关项的近似不动点作用量的展开系数。

Let us illustrate the above classification with the example of the Litim-Sannino (LS) model [172-176]. In its vanilla version, the LS model is a gauge-Yukawa model that contains gauge fields in the $\mathrm{SU}(N_c)$ gauge group, N_f fermions in the fundamental representation, and an uncharged complex scalar field, which forms an $N_f \times N_f$ matrix field. The theory is considered in the Veneziano limit where N_f and N_c are sent to infinity in a constant ratio. The beta function then develops an interacting UV fixed point that is proportional to the Veneziano parameter $\varepsilon = N_f/N_c - 11/2$. Choosing an infinitesimal value of ε allows to have a UV fixed point that is under full perturbative control.

我们以利蒂姆-桑尼诺 (LS) 模型为例说明上述分类 [172-176]。基础版 LS 模型是一个规范-汤川模型，包含 $\mathrm{SU}(N_c)$ 规范群下的规范场，基础表示下的 N_f 个费米子，以及一个构成 $N_f \times N_f$ 矩阵场的不带电复标量场。该理论在威尼齐亚诺极限下讨论，其中 N_f 和 N_c 按固定比值趋于无穷。β 函数随后会产生一个与威尼齐亚诺参数 $\varepsilon = N_f/N_c - 11/2$ 成正比的相互作用紫外不动点。取 ε 为无穷小量即可得到一个完全处于微扰控制下的紫外不动点。

In the LS model, the set of classical vertices is clearly defined: it contains all propagators generated by the kinetic terms and the Yang-Mills action, as well as the classical gluon and gluon-fermion vertices, the Yukawa vertex, and the two four-scalar vertices. It is important to note that higher-order interactions such as the four-Fermi vertex are not vanishing at the UV fixed point, but nonetheless, these vertices are part of Γ_{qu} since they belong to irrelevant operators at the UV fixed point. While the classification of relevant and irrelevant vertices is straightforward in the LS, the situation in gravity is far more intricate.

在 LS 模型中，经典顶点集定义清晰: 它包含由动能项和杨-米尔斯作用量生成的所有传播子，以及经典胶子顶点、经典胶子-费米子顶点、汤川顶点和两个四标量顶点。需要注意的是，四费米顶点这类高阶相互作用在紫外不动点处并不为零，但即便如此，这类顶点仍属于 Γ_{qu}，因为它们在紫外不动点处是无关算符。LS 模型中相关顶点与无关顶点的分类十分直接，而引力中的情况要复杂得多。

We proceed with the discussion of the subdominance of higher-order vertices $\Gamma_{\mathrm{qu}}^{(0,n)}$ that is tantamount to extracting the small parameter of the expansion scheme. First, we remark that the correlation functions $\Gamma_{\mathrm{qu}}^{(0,n)}$ are local in position space. However, they are not point-like (ultralocal). Hence, generic quantum vertices carry the same space-time and momentum dependences as the distribution parts:

我们接下来讨论高阶顶点 $\Gamma_{\rm qu}^{(0,n)}$ 的次主导性，这相当于提取展开方案的小参数。首先，我们注意到关联函数 $\Gamma_{\rm qu}^{(0,n)}$ 在位置空间是局域的，但并非点态 (超局域)。因此，一般量子顶点与分布部分具有相同的时空和动量依赖关系:

$$\lambda^{(\rm cl,dis)} \to \lambda^{(\rm fp,dis)}, \tag{74}$$

of the dressing of the fixed point-parts of the effective action; see (68) and (69). This is not surprising as the distribution part belongs to the quantum part of the dressing of a relevant fixed-point tensor structure. Hence, (69) generalizes to all quantum dressings:

即有效作用量中不动点部分的修饰；参见式 (68) 和 (69)。这并不出人意料，因为分布部分属于相关不动点张量结构修饰的量子部分。因此，式 (69) 可推广至所有量子修饰:

$$\lim_{p\to\infty} \lambda_{0,n}^{(\rm qu)}(\mathbf{p}_n)\Big|_{p_i^2=p^2} = 0. \tag{75}$$

Moreover, we assume that the dressings do not contain poles in momentum states or close massless resonances. This assumption entails that the dressings of the quantum parts of given vertices have a finite scattering length:

此外，我们假设修饰不包含动量态的极点或近零质量共振。该假设意味着给定顶点量子部分的修饰具有有限散射长度:

$$\lim_{\|\mathbf{x}_n\|\to\infty} \lambda^{(i)}_{\phi_{a_1}\cdots\phi_{a_n}}(\mathbf{x}_n) \leq e^{-\left(\xi^{(i)}_{\phi_{a_1}\cdots\phi_{a_n}}+\varepsilon_+\right)\|\mathbf{x}_n\|} \tag{76}$$

with a small positive $\varepsilon_+ \to 0$.

其中 $\varepsilon_+ \to 0$ 为小正数。

The simplest example of classical and quantum tensor structures is at the hand of the graviton propagator, which contains five tensor structures of which one is transverse traceless. The Einstein-Hilbert part features two tensor structures, namely, the transverse-traceless one and the physical scalar mode. The results in asymptotically safe gravity indicate that these two tensor structures are also that of the fixed-point action, and hence constitute $\Gamma_{\rm fp}^{(0,2)}$, as the other potentially relevant directions in the effective action related to the R^2 and $R_{\mu\nu}^2$ tensor structure do not generate the additional three tensor structures. Accordingly, the $\Gamma_{\rm qu}^{(0,2)}$ are triggered by the gauge fixing: they are only present for ensuring the invertibility of $\Gamma^{(0,2)}$. Note, however, that the dressings of $\Gamma_{\rm fp}^{(0,2)}$ also contain quantum parts $\lambda^{(\rm tt,dis)}$ and $\lambda^{(\rm s,dis)}$ as defined in (68),(69), and (74). Here,"tt" and "s" stand for the traceless-transverse and scalar part, respectively.

经典和量子张量结构最简单的例子可以参考引力子传播子，它包含五种张量结构，其中一种是横向无迹张量结构。爱因斯坦-希尔伯特部分具有两种张量结构，即横向无迹结构和物理标量模式。渐近安全引力的结果表明，这两种张量结构同样也是不动点作用量的张量结构，因此构成了 $\Gamma_{\mathrm{fp}}^{(0,2)}$ ，因为有效作用量中与 R^2 和 $R_{\mu\nu}^2$ 张量结构相关的其他潜在相关方向不会生成额外的三种张量结构。因此，$\Gamma_{\mathrm{qu}}^{(0,2)}$ 由规范固定触发: 它们的存在只是为了保证 $\Gamma^{(0,2)}$ 的可逆性。但需要注意，$\Gamma_{\mathrm{fp}}^{(0,2)}$ 的修正也包含量子部分 $\lambda^{(\mathrm{tt,dis})}$ 和 $\lambda^{(\mathrm{s,dis})}$ ，其定义见式 (68)、(69) 和 (74)。此处“tt”和“s”分别代表无迹横向部分和标量部分。

At the next order, the graviton three-point function contains 33 tensor structures of which seven are completely transverse-traceless [117]. One would need to diagonalize the tensor basis in order to determine how many of these tensor structures belong to $\Gamma_{\mathrm{cl}}^{(0,3)}$. Moreover, the dressings of the seven transverse-traceless tensor structures are not all independent but are related by diffeomorphism constraints. This has to be taken into account within a relevance analysis, which is briefly discussed in section ”Fixed Point and Critical Exponents.”

在下一阶，引力子三点函数包含 33 种张量结构，其中七种是完全横向无迹的 [117]。我们需要对张量基进行对角化，才能确定其中有多少种张量结构属于 $\Gamma_{\mathrm{cl}}^{(0,3)}$ 。此外，这七种横向无迹张量结构的修正并非全部独立，而是由微分同胚约束相互关联。这一点必须在相关性分析中加以考虑，我们将在“不动点与临界指数”一节中简要讨论。

A key property for apparent convergence is momentum locality. Momentum locality states that the flow of an n -point function decays faster than the n -point function itself in the limit of all momenta going to infinity:

表观收敛的一个关键性质是动量局域性。动量局域性指出，在所有动量趋于无穷大的极限下，n 点函数的流比 n 点函数本身衰减得更快:

$$\lim_{p\to\infty} \frac{\left|\partial_t \Gamma^{(n)}(\mathbf{p}_n)\right|}{\left|\Gamma^{(n)}(\mathbf{p}_n)\right|} = 0, \tag{77}$$

where the absolute value indicates the projection on some tensor structure. The limit should be understood such that all momenta p_i go to infinity. In quantum gravity, this was first investigated in [111, 113] where (77) was shown to hold for the transverse-traceless part of the graviton two- and three-point function; see also Fig. 3. Remarkably, the property holds for all gauge fixings and all momentum configurations as long as all external momenta are sent to infinity. The property does not hold for the transverse-traceless graviton four-point function and the scalar mode of the graviton [31, 114, 115] , which was associated with the overlap of the relevant R^2 direction. This intricate situation deserves a more systematic analysis.

其中绝对值表示对某个张量结构的投影。该极限应理解为所有动量 p_i 都趋于无穷大。在量子引力中，这一性质最早在文献 [111, 113] 中得到研究，研究表明式 (77) 对引力子两点和三点函数的横向无迹部分成立；另见图 3。值得注意的是，只要所有外动量都被送至无穷大，该性质对所有规范固定和所有动量构型都成立。该性质对横向无迹引力子四点函数和引力子 [31, 114, 115] 的标量模式不成立，这与相关 R^2 方向的重叠相关。这种复杂情况值得更系统的分析。

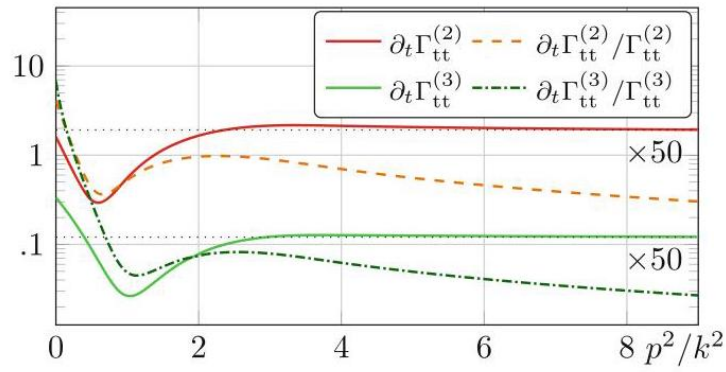


Fig. 3 Flow of the transverse-traceless graviton two- and three-point function, $\partial_t \Gamma^{(2)}_{\rm tt}$ and $\partial_t \Gamma^{(3)}_{\rm tt}$, as well as the flows weighted by the n -point function itself $\partial_t \Gamma^{(2)}_{\rm tt}/\Gamma^{(2)}_{\rm tt}$ and $\partial_t \Gamma^{(3)}_{\rm tt}/\Gamma^{(3)}_{\rm tt}$. Both flows are momentum local. The graviton three-point function is displayed at the momentum-symmetric point, but the momentum locality holds at general non-singular momentum configurations. (Figure adapted from [113])

图 3 横向无迹引力子两点函数和三点函数的流，即 $\partial_t \Gamma^{(2)}_{\rm tt}$ 和 $\partial_t \Gamma^{(3)}_{\rm tt}$ ，以及由 n 点函数本身加权后的流 $\partial_t \Gamma^{(2)}_{\rm tt}/\Gamma^{(2)}_{\rm tt}$ 和 $\partial_t \Gamma^{(3)}_{\rm tt}/\Gamma^{(3)}_{\rm tt}$ 。两种流都是动量局域的。引力子三点函数展示的是动量对称点的结果，但动量局域性在一般非奇异动量构型下也成立。(图改编自 [113])

Fixed Point and Critical Exponents

不动点与临界指数

Metric quantum gravity in terms of an asymptotically safe quantum field theory requires an occurrence of a nontrivial UV fixed point, the Reuter fixed point, with a few relevant directions in coupling space $\mathbf{g} = (g_1, g_2, \ldots)$, where all couplings are made dimensionless with the appropriate rescaling with powers of the RG-scale k . In the fluctuation approach, these couplings are typically expressed in terms of avatars of the cosmological constant, the Newton coupling (see (71)) and other couplings. But also fixed points of entire correlation functions [92, 111-115] and potentials [177] have been determined.

作为渐近安全量子场论的度量量子引力要求存在非平凡紫外不动点——罗伊特不动点，其在耦合空间中存在少量相关方向 $\mathbf{g} = (g_1, g_2, \ldots)$ ，其中所有耦合都通过 RG 标度幂次的适当重标度化为无量纲量 k 。在涨落方法中，这些耦合通常以宇宙常数化身、牛顿耦合化身 (见 (71)) 及其他耦合的形式表示。此外，已有研究确定了全关联函数 [92, 111-115] 和势 [177] 的不动点。

In terms of avatars of the Newton coupling, the most extensive study was done in [91] where five avatars were considered: the three-graviton vertex G_h , the graviton-ghost vertex G_c , the graviton-scalar vertex G_φ , the graviton-fermion vertex G_ψ , and the graviton-gluon vertex G_A . All of these avatars were found to have very similar beta functions and fixed-point values:

关于牛顿耦合化身，文献 [91] 完成了最全面的研究，共考虑了五个化身: 三引力子顶点 G_h 、引力子-鬼顶点 G_c 、引力子-标量顶点 G_φ 、引力子-费米子顶点 G_ψ 以及引力子-胶子顶点 G_A 。所有这些化身的 β 函数和不动点数值都非常相似:

$$\left(G_h^*, G_c^*, G_\varphi^*, G_\psi^*, G_A^*\right) = (0.58, 0.55, 0.74, 0.74, 0.84)\,. \tag{78}$$

This is a remarkable feature since these avatars are by no means identical. Instead, they are related by nontrivial STIs as explained in section "Background Independence and Symmetry Identities." The fact that they display this similarity was called effective universality [91,120] and indicates the near-perturbativeness of the interacting UV fixed point.

这是一个值得注意的特征，因为这些化身绝非完全相同。正如“背景独立性与对称性恒等式”章节所述，它们通过非平凡 Slavnov-Taylor 恒等式相互关联。这些化身展现出这种相似性的特点被称为有效普适性 [91,120]，表明相互作用紫外不动点近似于微扰。

In such a system with multiple avatars of one physical coupling, one needs to ask the question of how the mixing of the operators affects the relevant directions at the fixed point. In general, the relevant directions are obtained through the critical exponents, which are the (negative) eigenvalues of the stability matrix:

在这种一个物理耦合对应多个化身的体系中，我们需要探究算符混合如何影响不动点处的相关方向。一般来说，相关方向由临界指数得到，临界指数是稳定性矩阵的 (负) 本征值:

$$\mathcal{B}_{ij} = \frac{\partial \beta_i}{\partial g_j}, \quad \text{with} \quad \beta_i = \partial_t g_i. \tag{79}$$

This analysis has been done so far in terms of the couplings extracted from the vertex dressings of the renormalization-group invariant vertices (64) and specifically $\overline{\Gamma}_{\rm cl}^{(0,n)}(p)$, evaluated at symmetric points p; see [114] for pure gravity,[91] for gravity-matter systems and [171] for SM interactions.

迄今为止，这类分析都是基于从重整化群不变顶点 (64) 的顶点修饰中提取的耦合开展的，具体而言是在对称点处求值的 $\overline{\Gamma}_{\rm cl}^{(0,n)}(p)$ p；纯引力相关分析见 [114]，引力-物质体系见 [91]，标准模型相互作用见 [171]。

Let us illustrate the general structure of a stability analysis in the presence of avatars within the simple toy example of a system with two couplings, $\mathbf{g} = (g_1, g_2)$, that resemble avatars of the Newton coupling. These couplings have the β -functions:

我们通过一个简单的玩具实例说明存在化身时稳定性分析的一般结构: 该实例包含两个耦合 $\mathbf{g} = (g_1, g_2)$，二者都类似牛顿耦合的化身，它们的 β 函数为:

$$\beta_1(\mathbf{g}) = 2g_1 - (4 + 2\delta_1)\, g_1^2 + 2\delta_1 g_1 g_2,$$

$$\beta_2(\mathbf{g}) = 2g_2 - (4 + 2\delta_2)\, g_2^2 + 2\delta_2 g_1 g_2. \tag{80}$$

Upon the identification $g_1 = g_2$, both beta functions reduce to $\beta = 2g - 4g^2$ with the simple fixed-point $g^* = 1/2$ and the critical exponent $\theta = -2$. In the combined system, the fixed points are still given by $\mathbf{g}^* = (1/2, 1/2)$ and the stability matrix reads

做识别 $g_1 = g_2$ 后，两个 β 函数都约化为 $\beta = 2g - 4g^2$，得到简单不动点 $g^* = 1/2$ 和临界指数 $\theta = -2$。在组合体系中，不动点仍满足 $\mathbf{g}^* = (1/2, 1/2)$，稳定性矩阵可写为

$$\mathcal{B}\left(\mathbf{g}^*\right) = \begin{pmatrix} -2 + \delta_1 & -\delta_1 \\ -\delta_2 & -2 + \delta_2 \end{pmatrix}. \tag{81}$$

The eigenvalues are

本征值为

$$\theta_{1,2} = \left(-2, -2 + \delta_1 + \delta_2\right). \tag{82}$$

The first eigenvalue is precisely the physical eigenvalue that was also found in the identified beta functions. The second eigenvalue is shifted by the parameters δ_1 and δ_2 and might generate a second spurious relevant direction. This scenario indeed closely resembles the situation in [91] where five avatars of the Newton coupling were investigated and six relevant directions were found at the fixed point. Presumably, five of these directions can be unified into one physically relevant direction after the usage of appropriate symmetry identities (mSTIs). Or put differently, the relevant directions can be used to fulfill the symmetry constraint by the mSTI.

第一个本征值正是我们从已识别 β 函数中得到的物理本征值。第二个本征值会被参数 δ_1 和 δ_2 偏移，可能产生第二个伪相关方向。该情景确实与文献 [91] 的情况高度相似: 文献 [91] 研究了牛顿耦合的五个化身，在不动点处找到了六个相关方向。推测其中五个方向可以通过应用适当的对称性恒等式 (修正 Slavnov-Taylor 恒等式) 统一为一个物理相关方向。换句话说，可以利用相关方向通过修正 Slavnov-Taylor 恒等式满足对称性约束。

Relation to Other Approaches

与其他方法的关系

The fluctuation approach within an expansion about the flat background is tantamount to an expansion about vanishing curvature and in particular a vanishing Ricci scalar $R = 0$. As discussed before at the beginning of section "Expansion Schemes and RG-Invariant Vertex Dressings," this expansion scheme is explicit or implicit to most applications within the background approximation: few works go beyond the second order in curvature invariants, and in those works, one sees a rather quick convergence of the results in terms of the higher powers in the curvature scalar R^n [178-181], or higher powers build out of the Riemann tensor, e.g., $\left(R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}\right)^n$ and $R\left(R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}\right)^n$ at even and odd orders of the curvature expansion [182- 184]. In most of these computations, the first three orders, e.g., R^0, R^1, R^2, have an overlap with UV-relevant operators [185, 186]. This is in reassuring agreement with the findings in the fluctuation approach in pure quantum gravity, [88,89,111- 114]. In some works, a fourth relevant direction was found [183, 184], which is a combination of other higher-order curvature invariants. These findings in the Taylor expansion are sustained within computations with a full $f(R)$ potential. In the background-field approximation, this has been done in [179-184, 187-193]; in the fluctuation approach with the additional inclusion of momentum dependences, this has been studied in [88, 89] and within a double expansion in momenta and background curvature in [194].

在平直背景下展开的涨落方法等价于围绕零曲率，尤其是零里奇标量 $R=0$ 的展开。正如我们此前在“展开方案与 RG 不变顶点修饰”一节开头所讨论的，这种展开方案在背景近似的大多数应用中或是明确或是隐含: 几乎没有工作超越曲率不变量的二阶，而在那些做到的工作中，结果随着曲率标量 R^n 的更高次幂 [178-181] 表现出相当快的收敛性，或是更高次幂由黎曼张量构造而成，例如曲率展开奇偶阶的 $\left(R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}\right)^n$ 和 $R\left(R_{\mu\upsilon\rho\sigma}R^{\mu\upsilon\rho\sigma}\right)^n$ [182-184]。在大多数这类计算中，前三个阶次 (例如 R^0, R^1, R^2) 与紫外相关算符存在重叠 [185, 186]。这与纯量子引力中涨落方法的所得结果 [88,89,111-114] 十分吻合，令人安心。部分研究还发现了第四个相关方向 [183, 184]，它是其他高阶曲率不变量的组合。这些泰勒展开的结论在含完整 $f(R)$ 势的计算中依然成立。背景场近似下的相关计算可见 [179-184, 187-193]；涨落方法中额外纳入动量依赖的相关研究可见 [88, 89]，动量与背景曲率双展开下的研究可见 [194]。

In summary, both schemes support and are based on an expansion in powers of curvature invariants. However, in direct comparison, the background approximation relies on yet another approximation, namely, (58). This can be also clearly seen in the leading order of the curvature expansion: we can apply the momentum projection scheme used in the fluctuation approach for projecting on the running of the cosmological constant and the curvature scalar in the background approximation, leading to the same results as the heat-kernel techniques commonly used; see also [31]. For higher-order curvature invariants, the application of the momentum projection scheme requires the construction of a full tensor basis of local invariants including invariants that involve covariant derivatives.

总而言之，两种方案都支持且基于曲率不变量的幂次展开。但直接对比来看，背景近似还依赖另一项近似，即式 (58)。这一点在曲率展开的领头阶也清晰可见: 我们可以将涨落方法中使用的动量投影方案应用于背景近似下宇宙常数和曲率标量跑动的投影，得到的结果与常用热核技术的结果一致；参见文献 [31]。对于高阶曲率不变量，应用动量投影方案需要构造包含协变导数不变量在内的局部不变量完整张量基。

Of the approaches utilizing the background-field approximation, the form factor approach [16, 106-110] is most closely related to the fluctuation approach. Both approaches prioritize full momentum dependences. The form factor approach expands in order of curvatures (not derivatives) and the fluctuation approach in the orders of the correlation functions. There are mappings between these expansions which are detailed below. The most significant difference is that the form factor approach utilizes the background-field approximation. At finite k , the breaking of diffeomorphism invariance (or rather the BRST symmetry) by the regulator makes the comparison difficult since the background-field approximation is at odds with the diffeomorphism symmetry as explained in section ”Wrap-Up.” At vanishing cutoff scale $k \to 0$, the relation to the form factor approach is more straightforward: the fluctuation approach uses a boundary condition that satisfies the STIs, and therefore the correlation functions can be mapped to a diffeomorphism- or rather BRST-invariant action. In terms of form factors, this action is written as

在所有采用背景场近似的方法中，形状因子方法 [16, 106-110] 与涨落方法的联系最为紧密。两种方法都优先考虑完整动量依赖。形状因子方法按曲率阶数 (而非导数阶数) 展开，涨落方法按关联函数量级展开。这两种展开之间存在映射关系，我们会在下文详细说明。二者最显著的区别是形状因子方法采用背景场近似。在有限 k 下，规范场被调节器破坏微分同胚不变性 (更准确说是 BRST 对称性)，导致对比变得困难，因为正如我们在“总结”一节解释的，背景场近似与微分同胚对称性相矛盾。在截止尺度 $k \to 0$ 为零时，涨落方法与形状因子方法的关系更直接: 涨落方法采用满足 Slavnov-Taylor 恒等式 (STI) 的边界条件, 因此关联函数可以映射到微分同胚不变 (更准确说是 BRST 不变) 的作用量。用形状因子表示，该作用量可写为

$$\Gamma[\bar{g}, h] = \int_x \left(\frac{2\Lambda - R}{16\pi G_{\mathrm{N}}} + R f_R(\Box) R + C_{\mu\nu\rho\sigma} f_C(\Box) C^{\mu\nu\rho\sigma} + \dots \right) + S_{\mathrm{gf}} + S_{\mathrm{gh}}.$$

(83)

Using the fluctuation approach, several correlation functions have been computed at vanishing cutoff scales, including the propagator [82,92] and the three-graviton vertex [92]. These can be matched in a one-to-one fashion to the form factors assuming that the diffeomorphism constraints from the STI (see (56)) have been implemented as boundary conditions. For example, in an expansion about the flat background $g_{\mu\upsilon} = \eta_{\mu\upsilon} + \sqrt{G_{\mathrm{N}}} h_{\mu\upsilon}$, the form factor f_C is directly related to the wave-function renormalization via the transverse-traceless graviton propagator:

利用涨落方法，我们已经在零截止尺度下计算了多个关联函数，包括传播子 [82,92] 和三引力子顶点 [92]。若已经将来自 STI 的微分同胚约束 (见式 (56)) 作为边界条件实施，这些关联函数就可以与形状因子一一对应。例如，在平直背景 $g_{\mu\upsilon} = \eta_{\mu\upsilon} + \sqrt{G_{\mathrm{N}}} h_{\mu\upsilon}$ 附近的展开中，形状因子 f_C 通过横迹引力子传播子直接与波函数重整化相关:

$$G_{h_{tt}h_{tt}} = \frac{32\pi}{Z_{h_{\mathrm{tt}}}(p)\, p^2} = \frac{32\pi}{p^2 + 32\pi G_{\mathrm{N}} p^4 f_C(p^2)}, \tag{84}$$

where we have used that on-shell the cosmological constant is vanishing on the flat background. In straight analogy, the form factor f_R is expressed through the wave-function renormalization of the physical scalar mode via its propagator:

其中我们利用了在壳平直背景下宇宙常数为零的性质。同理，形状因子 f_R 通过物理标量模传播子的波函数重整化表示为:

$$G_{h_s h_s} = \frac{-16\pi}{Z_{h_{\mathrm{s}}}(p)\, p^2} = \frac{-16\pi}{p^2 - 96\pi G_{\mathrm{N}} p^4 f_R(p^2)}. \tag{85}$$

The scalar mode is negative as expected. This does not imply that gravity has a ghost mode: the negative massless excitation is non-propagating since it cancels out with parts of the transverse-traceless mode. We also emphasize that the wave-function renormalizations $Z_{h_{\mathrm{tt}}}$ and $Z_{h_{\mathrm{s}}}$ produce form factors f_R and f_C that agree exactly with one-loop computations for small values of p^2 [92, 195] . Specifically, the prefactors of the $\log(p^2)$ terms in f_R and f_C are scheme independent but gauge-dependent and these prefactors match exactly; see also section ”Lorentzian Quantum Gravity.” The scheme independence of the contributions is seen in the fRG computation through the fact that they are related to a p^4 derivative, which results in dimensionless flows.

标量模式如预期为负值。这不意味着引力存在鬼模式: 该负质量激发是非传播的，因为它会与横向无迹模式的部分相互抵消。我们还要强调，波函数重整化 $Z_{h_{\mathrm{tt}}}$ 和 $Z_{h_{\mathrm{s}}}$ 产生的形状因子 f_R 和 f_C 与 p^2 [92, 195] 取小值时的单圈计算完全一致。具体而言，f_R 和 f_C 中 $\log\left(p^2\right)$ 项的前置因子是 scheme 无关但规范依赖的，且这些前置因子完全匹配；参见“洛伦兹量子引力”一节。在泛函重整化群计算中，这些贡献的 scheme 无关性体现为: 它们与 p^4 导数相关，由此得到无量纲流。

The transverse-traceless three-graviton vertex overlaps with the cosmological constant, the Newton coupling, and the f_C form factor. These are however already fixed by lower-order correlation functions. The new overlap is with the Goroff-Sagnotti term and associated form factor:

横向无迹三引力子顶点与宇宙学常数、牛顿耦合以及 f_C 形状因子存在重叠。不过这些都已由低阶关联函数确定。新的重叠出现在戈罗夫-萨尼奥蒂项及相关形状因子中:

$$\Gamma_{\mathrm{GS}}[\bar{g}, h] = \int_x f_{C^3}\left(\nabla_1, \nabla_2, \nabla_3\right) C_{\mu\nu\rho\sigma} C^{\rho\sigma\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}, \tag{86}$$

where ∇_i only acts on the i th Weyl tensor. The transverse-traceless graviton three-point function was so far computed at the momentum-symmetric point at $k = 0$ [92], which fixes the form factor f_{C^3} at the momentum-symmetric point. This mapping can be systematically extended; see also [114] for an extensive discussion of overlaps between correlation functions and curvature operators.

其中 ∇_i 仅作用于第 i 个外尔张量。截至目前，横向无迹引力子三点函数仅在 $k = 0$ 的动量对称点处完成计算 [92]，该计算确定了形状因子 f_{C^3} 在动量对称点处的值。这种映射可以系统推广；关联函数与曲率算符之间重叠的详细讨论也可参见文献 [114]。

This concludes our discussion of the fundamentals of the fluctuation approach. We turn now to recently obtained results and refer to [31] for a more detailed overview.

至此我们完成了对涨落方法基础的讨论。接下来我们转向近期得到的结果，更详细的综述可参见文献 [31]。

Momentum-Dependent Correlation Functions

动量依赖关联函数

In the present section, we discuss the computation of momentum-dependent correlation functions, which constitute the backbone of the fluctuation approach. In the past decade, significant progress was achieved, and momentum dependencies up to the graviton four-point function, at vanishing cutoff scales, and even on Lorentzian backgrounds, were computed; see [82, 91, 92, 111-115, 120, 196, 197].

在本节中，我们讨论动量依赖关联函数的计算，这类关联函数是涨落方法的核心。过去十年，该领域已经取得了重大进展，直至引力子四点函数的动量依赖项都已在消失截断标度下，甚至洛伦兹背景上完成了计算；参见 [82, 91, 92, 111-115, 120, 196, 197]。

Momentum Dependence, Cutoff Dependence, and Diffeomorphism Invariance

动量依赖、截断依赖与微分同胚不变性

Physical correlation functions are obtained at vanishing cutoff scale, $k \to 0$, where all quantum fluctuations have been integrated out. At finite cutoff scales, the nonvanishing regulator may diffuse or even spoil important physics properties: while in many cases the cutoff dependence can be interpreted as an average momentum dependence at the symmetric point $p^2 = k^2$, this analogy is sometimes deceiving.

物理关联函数是在 vanishing 截断尺度 $k \to 0$ 下得到的，此时所有量子涨落都已被积分掉。在有限截断尺度下，非零的正则化项会扩散甚至破坏重要的物理性质: 尽管在很多情况下，截断依赖可以被解释为对称点 $p^2 = k^2$ 处的平均动量依赖，但这种类比有时具有误导性。

A prominent example in asymptotically safe gravity for the latter is the interpretation of the constant part of the cutoff-dependent effective action as the cosmological constant, which evidently does not have a momentum dependence; for a detailed discussion, see [114]. Here we just mention that the cosmological constant is commonly defined by the $\int d^4x\sqrt{g}$ part of the flow. However, this part of the flow is simply the normalization of the functional integral of metric gravity just as the constant part in any ordinary quantum field theory. Hence, it does not relate to a physics observable even at $k = 0$ in contrast to the flow of $\int d^4x\sqrt{g}$. In turn, the mass parameter of the graviton indeed is physical at $k = 0$ and is not directly related to the normalization of the functional integral. Moreover, the respective Nielsen identity derived from (53), which relates these two parameters, has a dominant regulator dependence at finite cutoff scales.

渐近安全引力中后者的一个典型例子，是将截断依赖的有效作用量的常数部分解释为宇宙学常数——显然宇宙学常数本身不依赖动量；详细讨论参见文献 [114]。在此我们仅指出，宇宙学常数通常由流的 $\int d^4x\sqrt{g}$ 部分定义。然而，这部分流只是度规引力泛函积分的归一化，就如同任何普通量子场论中的常数部分一样。因此，和 $\int d^4x\sqrt{g}$ 的流不同，即使在 $k = 0$ 处它也不对应物理可观测量。反过来，引力子的质量参数在 $k = 0$ 处确实是物理量，且和泛函积分的归一化没有直接关系。此外，由 (53) 导出的联系这两个参数的对应尼尔森恒等式，在有限截断尺度下存在显著的正则化依赖。

It is therefore of paramount importance to disentangle momentum and RG-scale dependence by integrating correlation functions from the UV fixed point to the IR while keeping full momentum dependence. The UV fixed point defines the starting point of the theory, and the relevant perturbations around this fixed point define the trajectory along which quantum fluctuations are integrated out. In [114], the system of coupled n -point functions has been solved up to the graviton four-point function. All n -point functions were evaluated at momentum-symmetric points with external transverse-traceless projections. A UV fixed point was found at

因此，从紫外不动点积分关联函数到红外的同时保留完整动量依赖，对于区分动量依赖和重整化群尺度依赖至关重要。紫外不动点定义了理论的起点，该不动点附近的相关扰动决定了量子涨落被逐步积分掉的轨迹。在文献 [114] 中，耦合的 n 点函数体系已求解到引力子四点函数。所有 n 点函数都在动量对称点处计算，并带有外源横向无迹投影。已找到紫外不动点位于

$$(\mu^*, \lambda_3^*, \lambda_4^*, g_3^*, g_4^*) = (-0.45, 0.12, 0.028, 0.83, 0.57)\,, \tag{87}$$

where g_n and λ_n are the dimensionless avatars of the Newton coupling and the cosmological constant, the latter corresponding momentum independent part of the graviton n -point function; for more details, see [114]. The graviton mass parameter $\mu = -2\lambda_2$ is the momentum-independent part of the graviton two-point function. The critical exponents of the fixed point are given by

其中 g_n 和 λ_n 分别是牛顿耦合和宇宙学常数的无量纲形式，后者对应引力子 n 点函数的动量无关部分；更多细节参见文献 [114]。引力子质量参数 $\mu = -2\lambda_2$ 是引力子两点函数的动量无关部分。该不动点的临界指数为

$$\theta_i = (4.7, 2.0 \pm 3.1i, -2.9, -8.0)\,, \tag{88}$$

where a positive sign corresponds to a UV-attractive direction. The three UV-attractive directions were associated with the operators or rather tensor structures $\sqrt{g}, \sqrt{g}R$, and $\sqrt{g}R^2$. In contrast, the operator $\sqrt{g}R_{\mu\nu}^2$ is not generated in the present approximation from the $\sqrt{g}, \sqrt{g}R$ tensor structures. The latter property was inferred from the momentum dependence of the graviton three- and four-point functions.

其中正号对应紫外吸引方向。三个紫外吸引方向分别对应算符 (更准确地说是张量结构) $\sqrt{g}, \sqrt{g}R$ 和 $\sqrt{g}R^2$ 。反之，在当前近似下，算符 $\sqrt{g}R_{\mu\nu}^2$ 不会由 $\sqrt{g}, \sqrt{g}R$ 张量结构生成。这一性质是从引力子三点和四点函数的动量依赖中推得的。

From this UV fixed point, there is a three-dimensional critical hypersurface that leads to the IR where one wants to match general relativity with the Einstein-Hilbert action and hence all avatars of the Newton coupling and the cosmological constant agree:

从该紫外不动点出发，存在一个三维临界超曲面通向红外，在红外处我们希望将理论与爱因斯坦-希尔伯特作用量形式的广义相对论匹配，因此牛顿耦合和宇宙学常数的所有无量纲形式满足:

$$G_i = \bar{G}_i = G,\ \Lambda_i = \overline{\Lambda}_i = \Lambda,\ \forall i \in \mathbb{N}. \tag{89}$$

Equation (89) should be obtained by first running to $k = 0$ and then setting $p = 0$. Evidently, (89) satisfies the NI (53) and the STI (56), and hence any trajectory leading to (89) constitutes a diffeomorphism-invariant theory. Note that (89) fixes two of the relevant parameters of the theory G, Λ in terms of an IR renormalization condition for the fluctuation couplings, whose flows provide the dynamical closed set of flow equations of the theory; see the discussion in section ”Flow Equations of Correlation Functions” and specifically (45). The third relevant direction in coupling space is related to the R^2 term, and its IR value can be set to zero on the level of the fluctuation couplings with a further fine-tuning condition that concerns the p^4 -part of the vertices.

式 (89) 应当先跑动到 $k = 0$, 再设定 $p = 0$ 得到。显然, (89) 满足尼尔森恒等式 (53) 和 Slavnov-Taylor 恒等式 (56)，因此任何通向 (89) 的轨迹都构成微分同胚不变的理论。注意 (89) 利用涨落耦合的红外重整化条件固定了理论 G, Λ 的两个相关参数，这些参数的流给出了该理论封闭的动力学流方程；参见“关联函数的流方程”一节的讨论，特别是 (45) 式。耦合空间中第三个相关方向和 R^2 项相关，它的红外值可以通过额外的精细调节条件在涨落耦合层面设定为零，该条件涉及顶点的 p^4 部分。

We emphasize that (89) implies a fine-tuning condition of order $n+m$ if n fluctuation avatars of G and m fluctuation avatars of Λ are considered in a given approximation: the initial conditions for these $n+m$ coupling parameters have to be tuned such that (89) holds at $k=0$ (and $p=0$). If (89) is obtained, the IR part of the full effective action (low momentum scales at $k=0$) is simply the Einstein-Hilbert action. Accordingly, solving these fine-tuning conditions at $k=0$ and $p=0$ is tantamount to solving the modified symmetry identities (89) and (53) in the given approximation. In turn, if (89) is not satisfied and other couplings and operators are absent in the IR, the theory is not diffeomorphism-invariant. A more detailed discussion can be found in [31].

我们强调，若在给定近似下考虑 n 个 G 涨落化身与 m 个 Λ 涨落化身，式 (89) 就暗含了一个阶为 $n+m$ 的微调条件: 这些 $n+m$ 个耦合参数的初始条件必须经过微调，使得式 (89) 在 $k=0$ (及 $p=0$) 处成立。若得到满足式 (89) 的结果，全有效作用量的红外部分 ($k=0$ 处的低动量标度) 就恰好是爱因斯坦-希尔伯特作用量。相应地，在 $k=0$ 和 $p=0$ 处求解这些微调条件，等价于在给定近似下求解修正的对称恒等式 (89) 和 (53)。反过来，若式 (89) 不满足，且红外区内不存在其他耦合与算符，该理论就不满足微分同胚不变性。更详细的讨论可参见文献 [31]。

In [114], many physically interesting UV-IR trajectories were found, including the example displayed in Fig. 4. On that trajectory, the Newton coupling $G(p)$ matches the physical value in the IR and the cosmological constant runs to a finite negative value. The fine-tuning condition for diffeomorphism invariance has been solved for the two avatars of the Newton coupling present in the approximation: $G=G_3 \approx G_4$ and for two avatars of the cosmological constant, $\Lambda_2 \approx \Lambda_3$. The coupling λ_4 was not fine-tuned as the higher λ_n are increasingly irrelevant and the fine-tuning effort increases substantially with the dimension of the fine-tuning manifold.

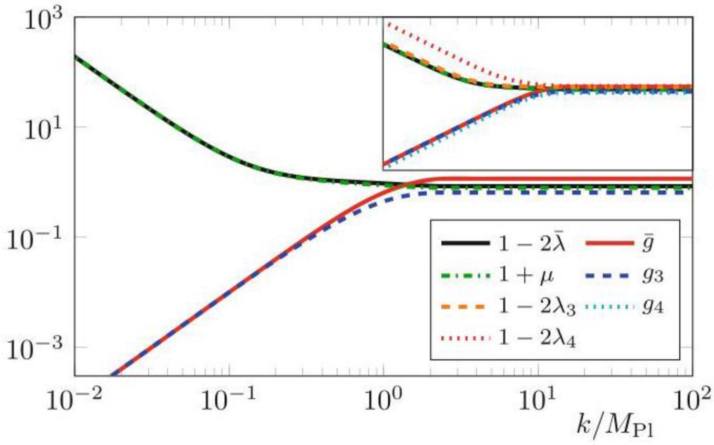
在文献 [114] 中，作者找到了多条物理上有意义的紫外-红外轨迹，包括图 4 所示的例子。在该轨迹上，牛顿耦合 $G(p)$ 在红外区匹配物理值，宇宙常数演化到有限负值。该近似中存在两个牛顿耦合化身，微分同胚不变性的微调条件已对其求解: $G=G_3 \approx G_4$ ，同时也对两个宇宙常数化身 $\Lambda_2 \approx \Lambda_3$ 完成了求解。耦合 λ_4 未做微调，因为更高阶的 λ_n 无关性越来越强，且微调难度会随微调流形的维度大幅提升。

With the diffeomorphism-consistent solution of the fluctuation system, the background couplings of the diffeomorphism-invariant background effective action can be computed. In Fig. 4, the UV-IR trajectories of the background cosmological constant and background Newton coupling with $G_3=G$ and $\bar{\Lambda}=\Lambda$ are displayed. Such a choice can always be achieved easily since the background couplings are non-dynamical and higher-order background couplings do not feed back into the flow of the lower-order ones; see the discussion in section ”Flow Equations of Correlation Functions” and specifically (46). Hence, there is no fine-tuning problem, whose difficulty is increasing with the dimension of the coupling manifold.

借助涨落系统满足微分同胚一致性的解，我们可以计算微分同胚不变背景有效作用量的背景耦合。图 4 展示了带有 $G_3=G$ 和 $\bar{\Lambda}=\Lambda$ 的背景宇宙常数与背景牛顿耦合的紫外-红外轨迹。由于背景耦合是非动力学的，且高阶背景耦合不会反向影响低阶耦合的流，因此这类选择总能轻松实现；参见章节“关联函数的流方程”中的讨论，特别是式 (46)。因此，这里不存在难度随耦合流形维度增加而升高的微调问题。

Fig. 4 UV-IR trajectory from the UV fixed point (87) to the IR with general relativity. Here, the Newton couplings are fixed to their classical value and the cosmological constant is finite and negative. Other values of the cosmological constant can be chosen as well. (The results are taken from [114])

图 4 从紫外不动点 (87) 到红外区广义相对论的紫外-红外轨迹。此处牛顿耦合被固定为经典值，宇宙常数为有限负值。也可以选择宇宙常数的其他值。(结果取自文献 [114])



This concludes our discussion of how to practically use the fluctuation approach for solving the flows of diffeomorphism-invariant metric quantum gravity within given approximations. We add a few further technical remarks on the coupling choices, the Nielsen (NI) and Slavnov-Taylor (STI) identities in the limit $k \to 0$ and apparent convergence as discussed in section ”Apparent Convergence”:

至此我们结束关于如何在给定近似内实际应用涨落方法求解微分同胚不变度量量子引力的流的讨论。我们补充若干进一步的技术说明，涉及耦合选择、$k \to 0$ 极限下的尼尔森恒等式 (NI) 和斯拉夫诺夫-泰勒恒等式 (STI)，以及“表观收敛”章节讨论的表观收敛:

Choice of Λ The choice $\Lambda < 0$ has been taken in order to avoid discussing the subtleties that come with the definition of the effective action as a Legendre transform: for non-convex classical action, the classical effective action is the convex hull of the classical action. In [114] also other values of the cosmological constant, including vanishing and positive values, have been considered.

Λ 的选择我们采用选择 $\Lambda < 0$ 是为了避免讨论有效作用量作为勒让德变换定义带来的微妙问题: 对于非凸经典作用量，经典有效作用量是经典作用量的凸包。文献 [114] 也考虑了宇宙常数的其他取值，包括零和正值。

Physical limit $k \to 0$ In general, the limits $k \to 0$ and $p \to 0$ do not commute, and the limit $k \to 0$ of $G_k(p=0)$ is in the regularized regime with nontrivial STIs and NIs even for $k = 0$. In [114] we still have chosen this order of limits as the couplings have been obtained by taking into account the full momentum dependence in the regime $0 \leq p^2 \lesssim k^2$. Hence, we expect subleading effects of the nontrivial symmetry identities to be present which have been ignored there.

物理极限 $k \to 0$ 一般而言，极限 $k \to 0$ 与极限 $p \to 0$ 不对易，即使对于 $k = 0$ ，$G_k(p=0)$ 的极限 $k \to 0$ 也处于带有非平凡 STI 和 NI 的正则化区域。在文献 [114] 中我们仍选择了这种极限顺序，因为耦合是通过考虑 $0 \leq p^2 \lesssim k^2$ 区域内的完整动量依赖得到的。因此我们预期，文中忽略的非平凡对称恒等式仅带来次领头阶效应。

Apparent convergence Finally, it has been studied in [114] how the results change if higher-order couplings such as g_4 and λ_4 are computed instead of fixed to their Einstein-Hilbert counterparts $\lambda^{(\mathrm{EH})}(p)$ as discussed in section ”Apparent Convergence.” We have interpreted the respective minimal changes of the lower-order avatars of the Newton coupling and cosmological constant as the onset of apparent convergence. These promising signatures award further studies in more advanced approximations.

表观收敛最后，文献 [114] 研究了: 若将 g_4 和 λ_4 这类高阶耦合进行计算，而非固定在“表观收敛”一节讨论的爱因斯坦-希尔伯特对应项 $\lambda^{(\mathrm{EH})}(p)$，结果会发生何种变化。我们将牛顿耦合与宇宙常数低阶形式对应的极小变化解读为表观收敛的开端。这些可观的迹象表明，值得在更先进的近似下开展进一步研究。

The results in [114] obtained by the fine-tuning procedure described above include momentum-dependent correlation functions and hence form factors in disguise. On a given UV-IR trajectory, the momentum-dependent correlation functions or rather their momentum-dependent flows can be integrated, resulting in momentum-dependent physical correlation functions at $k = 0$. This integration on the basis of the momentum-dependent flows in [114] was performed in [92] for the transverse-traceless propagator mode and the transverse-traceless three-graviton vertex at the symmetric momentum point. There, a UV-IR trajectory with a vanishing IR cosmological constant was chosen. The results are displayed in Fig. 5.

文献 [114] 中通过上述微调过程得到的结果包含动量依赖关联函数，因此本质上是隐含的形状因子。在给定的紫外-红外轨迹上，动量依赖关联函数 (更准确地说是其动量依赖流) 可以积分，得到 $k = 0$ 处的动量依赖物理关联函数。文献 [114] 中基于动量依赖流的这项积分工作由文献 [92] 针对对称动量点处的横向无迹传播子模式与横向无迹三引力子顶点完成，该工作选取了红外宇宙常数为零的紫外-红外轨迹，结果展示在图 5 中。

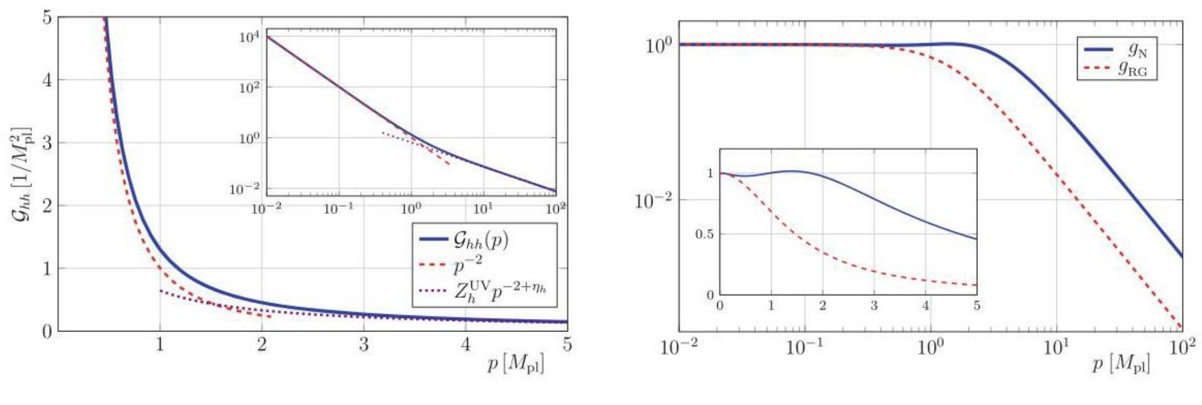


Fig. 5 We show the physical momentum dependence of the propagator (left panel) and the Newton coupling (right panel). The propagator scales with $1/p^2$ in the IR and with $1/p^{2-\eta_h^*}$ in the UV where η_h^* is the fixed-point anomalous dimension. The crossover regime is around the Planck scale. In the right panel, we show a comparison between the physical Newton coupling, $g_{\mathrm{N}}(p) = G_{\mathrm{N}}(p) M_{\mathrm{pl}}^2$, and the RG improved Newton coupling, $g_{\mathrm{RG}} = g_{k=p}$, both in units of the Planck mass. While the functions agree qualitatively, the physical Newton coupling is needed for a quantitative understanding. (Figure adapted from [92])

图 5 我们展示了传播子 (左图) 与牛顿耦合 (右图) 的物理动量依赖性。传播子在红外区标度为 $1/p^2$ ，在紫外区标度为 $1/p^{2-\eta_h^*}$ ，其中 η_h^* 是不动点反常维度。交叉区位于普朗克尺度附近。右图中我们对比了物理牛顿耦合 $g_{\mathrm{N}}(p) = G_{\mathrm{N}}(p) M_{\mathrm{pl}}^2$ 和 RG 改进牛顿耦合 $g_{\mathrm{RG}} = g_{k=p}$ ，二者均以普朗克质量为单位。虽然二者在定性上一致，但定量理解仍需使用物理牛顿耦合。(图改编自文献 [92])

The left panel shows the momentum dependence of the propagator in units of the Planck scale M_{pl} . Below the Planck scale, the propagator scales with $1/p^2$ and has a subleading logarithmic contribution that matches exactly the effective field theory results [92, 195]. This is discussed in more detail in section ”Lorentzian Quantum Gravity.” There is a crossover regime around the Planck scale, and in the trans-Planckian regime for $p/M_{\mathrm{pl}} \to \infty$ the propagator scales with $1/p^{2-\eta_h^*}$, where η_h^* is the fixed-point anomalous dimension. In this truncation, the fixed-point anomalous dimension was found to be $\eta_h^* \approx 1.03$, implying that the fluctuation graviton propagator scales approximately with $1/p$ in the UV. This propagator was analytically continued and used to reconstruct the graviton spectral function from the displayed Euclidean data. More details are provided in section ”Lorentzian Quantum Gravity,” where the results of the direct Lorentzian computations are also presented.

左图展示了以普朗克尺度 M_{pl} 为单位的传播子动量依赖性。普朗克尺度以下，传播子标度为 $1/p^2$ ，并带有次领先对数贡献，与有效场论结果 [92, 195] 完全吻合，这一点在“洛伦兹量子引力”一节有更详细的讨论。传播子在普朗克尺度附近存在交叉区，对于 $p/M_{\mathrm{pl}} \to \infty$ 对应的跨普朗克区，传播子标度为 $1/p^{2-\eta_h^*}$ ，其中 η_h^* 是不动点反常维度。在该截断下，求得不动点反常维度为 $\eta_h^* \approx 1.03$ ，说明涨落引力子传播子在紫外区近似标度为 $1/p$ 。该传播子已做了解析延拓，并被用于从给出的欧几里得数据重构引力子谱函数。更多细节详见“洛伦兹量子引力”一节，该节还展示了直接洛伦兹计算的结果。

The right panel in Fig. 5 shows the momentum dependence of the physical Newton coupling at vanishing cutoff, $g_{\mathrm{N}}(p) = G_{\mathrm{N}}(p) M_{\mathrm{pl}}^2$, which was extracted from the momentum dependence of the graviton three-point function. It is compared to the RG improved Newton coupling where the RG scale is identified with the momentum, $g_{\mathrm{RG}} = g_{k=p}$. In the IR, both couplings are constant and agree exactly as they are both fixed to the classical value. Beyond the Planck scale, both couplings scale with $1/p^2$ which implies that their dimensionless counterpart takes the constant fixed-point value. Hence, both couplings have qualitatively the same features while they quantitatively clearly differ. These results support that RG improvement can be used qualitatively, at least in a system with only one physical scale. The only physical scale here is the momentum of the vertex at the symmetric point. It is expected that the RG improvement works less well with multiple scales, e.g., away from the momentum-symmetric point.

图 5 的右面板展示了截断消失时物理牛顿耦合的动量依赖关系 $g_{\mathrm{N}}(p) = G_{\mathrm{N}}(p) M_{\mathrm{pl}}^2$ ，该关系从引力子三点函数的动量依赖中提取得到。我们将其与 RG 标度等同于动量的 RG 改进牛顿耦合 $g_{\mathrm{RG}} = g_{k=p}$ 进行了比较。在红外区域，两种耦合均为常数且完全一致，因为二者都固定在经典值上。超出普朗克标度后，两种耦合都随 $1/p^2$ 标度变化，这意味着它们的无量纲形式取恒定不动点值。因此，两种耦合在定性上特征一致，在定量上则存在明显差异。这些结果支持 RG 改进可用于定性分析，至少在仅含一个物理标度的系统中成立。此处唯一的物理标度是对称点顶点的动量。可以预期，在存在多个标度的情形下，例如偏离动量对称点时，RG 改进的效果会变差。

Scattering Amplitudes

散射振幅

We close this section with a discussion on the relation of the results on vertex dressings and the respective RG-invariant couplings or form factors to matrix elements of the S -matrix and the respective form factors of the S -matrix. To that end, we consider 2-to-2 graviton scattering in the tt-channel and explain its relation to the momentum-dependent Newton coupling $G_{\mathrm{N}}(p)$. The lowest-order tree-level diagram for this matrix element is depicted in Fig. 6, where the dashed lines indicate the transverse-traceless contraction of the external legs that is done with the tt-projection operator. The transition amplitude $\mathcal{M}_{gg\to gg}$ of this process is nothing but

我们在本节最后讨论顶点修饰、相应的重整化群不变耦合或形状因子的结果与 S 矩阵矩阵元以及 S 矩阵相应形状因子之间的关系。为此，我们研究 tt 道中的 2 对 2 引力子散射，阐释它与动量依赖的牛顿耦合 $G_{\mathrm{N}}(p)$ 的关联。该矩阵元的最低阶树图如图 6 所示，其中虚线表示外腿通过 tt 投影算子完成的横向无迹收缩。该过程的跃迁振幅 $\mathcal{M}_{gg\to gg}$ 即为

$$\mathcal{M}_{gg\to gg} \simeq p^2 G^{(tt)}_{h_b h_a}(p)\left[\Gamma^{(h_a h_b h_c)}(p)\, G_{h_c h_d}(p)\, \Gamma^{\left(h_d h_e h_f\right)}(p)\right] p^2 G^{(tt)}_{h_f h_e}(p)\,,$$

(90)

where $a, \ldots, f$ in (90) are the indices of second rank Lorentz tensors, for example, $a = \mu\nu$, and hence all Lorentz indices in (90) are contracted. We remark that in (90) we evaluated the transition amplitude at the symmetric point $p_i^2 = p^2$ for a general p for the sake of simplicity, and hence the external gravitons are not on-shell. The external dressings $p^2 G_{hh}$ accommodate the tt -projection as well as the wave function factors $1/Z_h^{1/2}\left(p_i^2\right)$ in the LSZ formula. At its core and in the approximation used here, (90) is nothing but the renormalization group-invariant three-graviton vertex squared, $\overline{\Gamma}^{(3)}_{hhh}$ defined in (64). This leads us to

式 (90) 中 $a, \ldots, f$ 是二阶洛伦兹张量的指标，例如 $a = \mu\nu$ ，因此 (90) 中所有洛伦兹指标都已完成收缩。需要说明，为简化推导，我们是在对称点 $p_i^2 = p^2$ 对一般 p 计算跃迁振幅，因此外引力子不在壳。外修饰项 $p^2 G_{hh}$ 包含了 tt 投影以及 LSZ 公式中的波函数因子 $1/Z_h^{1/2}\left(p_i^2\right)$。就核心性质和本文所用近似而言，式 (90) 本质就是重整化群不变三引力子顶点的平方，即 (64) 式定义的 $\overline{\Gamma}^{(3)}_{hhh}$ 。由此我们得到

$$\mathcal{M}_{gg\to gg} \propto p^2 G_3(p)\,, \tag{91}$$

which entails that the matrix element goes to a constant for large momenta and that the form factor of the transition amplitude is nothing but the form factor squared of the three-graviton vertex. Note that strictly speaking such a 2-to-2 graviton scattering process is not physical, as a state with n -gravitons is not gauge independent. In any case, (91) or similar scattering processes for (physical) matter fields which are transmitted by gravitons have to be evaluated on-shell. Then the external momenta $\left(p_i^{(\mathrm{ext})}\right)^2 = m_i^2$, where m_i are the pole masses of the respective particles or fields. Then, the process (Fig. 6) constitutes an s -channel scattering process with $s = p^2$, and the coupling $G(p)$ in (91) generalizes to a renormalization group-invariant form factor $\overline{\lambda}_{hhh}(p_1, p_2, p)\,\overline{\lambda}_{hhh}(-p, -p_3, -p_4)$, where all momenta are counted incoming; see also the discussion in [19].

由此可知，大动量下矩阵元趋近于常数，跃迁振幅的形状因子本质就是三引力子顶点的形状因子平方。请注意，严格来说这类 2 对 2 引力子散射过程不具有物理意义，因为含 n 个引力子的态不满足规范无关性。无论如何，(91) 式或由引力子传递的 (物理) 物质场的类似散射过程都需要在壳计算。此时外动量满足 $\left(p_i^{(\text{ext})}\right)^2 = m_i^2$，其中 m_i 是对应粒子或场的极点质量。因此，图 6 所示过程构成了 s 道散射，满足 $s = p^2$，(91) 式中的耦合 $G(p)$ 可推广为重整化群不变形状因子 $\bar{\lambda}_{hhh}(p_1, p_2, p)\bar{\lambda}_{hhh}(-p, -p_3, -p_4)$，其中所有动量都按入射计数；另见文献 [19] 中的讨论。

Fig. 6 Tree-level 2-to-2 graviton scattering diagram in the tt-channel. The dashed lines indicate the transverse-traceless contraction of the external legs; see (90). (Figure adapted from [92])

图 6 tt 道树图级 2 对 2 引力子散射费曼图。虚线表示外腿的横向无迹收缩，参见式 (90)。(图改编自文献 [92])

Outlook

展望

The results on momentum-dependent correlation functions reviewed in this section allow for the resolution of asymptotically safe gravity and asymptotically safe gravity-matter systems in a systematic expansion scheme. This scheme is controlled by apparent convergence and allows for the resolution of the underlying Slavnov-Taylor identities at vanishing cutoff scales which guarantee physical diffeomorphism invariance.

本节综述的动量依赖关联函数研究结果，可在系统展开框架中求解渐近安全引力与渐近安全引力-物质系统。该框架由表观收敛控制，能够在 vanishing cutoff 尺度下求解底层的斯拉夫诺夫-泰勒恒等式，该恒等式保证了物理微分同胚不变性。

The Asymptotically Safe Standard Model

渐近安全标准模型

The results obtained in the fluctuation approach for pure gravity, including that on apparent convergence discussed in section ”Momentum-Dependent Correlation Functions,” can be put to work in a general gravity-matter system. This line of study was initiated in [198] and by now includes the study of the fixed-point structure of general gravity-matter theories with scalars, fermions, and gauge fields. The general interplay of gravity and matter in the background-field approximation, hybrid approaches, and the fluctuation approach including studies beyond the minimal coupling approximation (see [59, 62, 65, 70, 89, 91, 120, 169, 170, 196 − 234]) was extensively reviewed in [17].

纯引力涨落方法得到的结果，包括“动量依赖关联函数”一节讨论的表观收敛结果，都可应用于一般引力-物质系统。这一研究方向由 [198] 开创，目前已涵盖对包含标量、费米子和规范场的一般引力-物质理论不动点结构的研究。背景场近似、混合方法以及包含超出最小耦合近似研究 (见 [59, 62, 65, 70, 89, 91, 120, 169, 170, 196 − 234]) 在内的涨落方法中引力与物质的普遍相互作用已在 [17] 中得到全面综述。

Fundamentals of the Asymptotically Safe Standard Model

渐近安全标准模型基础

In the present section, we focus on the recent milestone of full non-perturbative SM flows including quantum gravity within the fluctuation approach achieved in [171]. There, the SM was embedded in asymptotically safe gravity within the advanced fluctuation approximation discussed in section ”Momentum-Dependent Correlation Functions.” In [171] the flows of the wave functions (96) of all SM fields and the graviton as well as the flows of the primitively divergent couplings (97) have been considered. The field content of the Asymptotically Safe Standard Model (ASSM) is given by

在本节中，我们重点介绍文献 [171] 在涨落方法框架下得到的最新里程碑成果: 包含量子引力的完整非微扰标准模型流。文中，我们在“动量依赖关联函数”一节讨论的进阶涨落近似下，将标准模型嵌入了渐近安全引力。文献 [171] 中研究了所有标准模型场与引力子波函数的流 (96)， 以及原生发散耦合的流 (97)。渐近安全标准模型 (ASSM) 的场内容如下:

$$\phi_{\text{grav}} = \left(h_{\mu\nu}, c_\mu, \bar{c}_\mu\right), \ \phi_{\text{SM}} = \left(\mathcal{A}_\mu, C, \bar{C}, l, \bar{l}, q, \bar{q}, \Phi\right). \tag{92}$$

The matter superfield ϕ_{SM} comprises the gauge and ghost fields of the SM gauge group $\text{U}(1)_\text{Y} \times \text{SU}(2)_\text{L} \times \text{SU}(3)_\text{C}$:

物质超场 ϕ_{SM} 包含标准模型规范群 $\text{U}(1)_\text{Y} \times \text{SU}(2)_\text{L} \times \text{SU}(3)_\text{C}$ 的规范场和鬼场:

$$\mathcal{A}_\mu = \left(B_\mu, A_\mu^a, G_\mu^b\right), \ \mathcal{C} = \left(c^a, c^b\right), \tag{93}$$

with the hypercharge gauge field B_μ , the weak gauge fields A_μ^a with $a = 1, 2, 3$, and the gluons G_μ^b with $b = 1, \ldots, 8$. The field C contains the respective ghost fields of the weak and strong gauge groups. ϕ_{SM} also contains the three families of quarks q and leptons l :

其中包括超荷规范场 B_μ ， 满足 $a = 1, 2, 3$ 的弱规范场 A_μ^a ， 以及满足 $b = 1, \ldots, 8$ 的胶子 G_μ^b 。场 C 包含弱规范群和强规范群对应的鬼场。ϕ_{SM} 还包含三夸克家族 q 和轻子家族 l :

$$q = (d, u, s, c, b, t), \ l = \left(e, \upsilon_e, \mu, \upsilon_\mu, \tau, \upsilon_\tau\right). \tag{94}$$

The scalar field Φ is the Higgs doublet, which is parametrized with

标量场 Φ 是希格斯二重态，参数化为

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ v + H + i\mathcal{G}_3 \end{pmatrix}, \; \rho = \mathrm{tr}\, \Phi^\dagger \Phi, \tag{95}$$

where v is the flowing minimum and H is the (radial) fluctuation Higgs field with a vanishing expectation value $\langle H \rangle = 0$, as the latter is explicitly carried by v . The $\mathcal{G}_i$ with $i = 1, 2, 3$ are the Goldstone modes.

其中 v 是流动极小值，H 是 (径向) 涨落希格斯场，其期望值 $\langle H \rangle = 0$ 为零，因为期望值由 v 显式承载。满足 $i = 1, 2, 3$ 的 $\mathcal{G}_i$ 是戈德斯通模。

We briefly discuss the approximation to the full effective action of the ASSM in terms of the flows taken into account in [171]. The flows have been computed self-consistently: all flowing parameters computed have been fed back into the flows. First of all, all fields have to be augmented with their cutoff-dependent wave functions:

我们结合文献 [171] 中考虑的流，简要讨论 ASSM 全有效作用量的近似方案。这些流是自洽计算得到的: 所有计算出的流动参数都被反馈回流中。首先，所有场都必须补充依赖截断的波函数:

$$Z_{\phi,k} = \left(Z_{A,k}, Z_{C,k}, Z_{l,k}, Z_{q,k}, Z_{\Phi,k}, Z_{h,k}, Z_{c,k} \right). \tag{96}$$

and the respective flows have been solved. Moreover, the flow of all primitively divergent couplings was considered:

并且相应的流已经得到求解。此外，文中还考虑了所有原生发散耦合的流:

$$\lambda_k = \left(g_{1,k}, g_{2,k}, g_{3,k}, y_{q,k}, y_{l,k}, \lambda_{\Phi,k}, G_k, \Lambda_k \right). \tag{97}$$

These couplings have been obtained from the RG-invariant dressings (66) by dividing the vertex dressings with the respective powers of the wave functions in (96).

这些耦合可从 RG 不变装扮 (66) 得到: 将顶点装扮项除以 (96) 中波函数的对应次幂。

The g_i with $i = 1, 2, 3$ are related to the hypercharge gauge coupling with $g_1 \equiv \sqrt{5/3} g_Y$, the weak gauge coupling g_2 , and the strong gauge coupling g_3 . The couplings y_q and y_l are the Yukawa couplings to the quarks and leptons, and λ_Φ is the quartic interaction of the Higgs doublet. In the Higgs sector, higher-order couplings were also considered: the Higgs potential was expanded in a high-order Taylor expansion around the flowing minimum. On the gravity side, the running of the Newton coupling G and the cosmological constant Λ were taken into account.

满足 $i = 1, 2, 3$ 的 g_i 分别对应满足 $g_1 \equiv \sqrt{5/3} g_Y$ 的超荷规范耦合、弱规范耦合 g_2 和强规范耦合 g_3 。耦合 y_q 和 y_l 是夸克和轻子的汤川耦合，λ_Φ 是希格斯二重态的四次相互作用。在希格斯区，文中还考虑了高阶耦合: 希格斯势在流动极小值附近做了高阶泰勒展开。引力方面，也考虑了牛顿耦合 G 和宇宙学常数 Λ 的跑动。

The couplings in (97) have different avatars depending on the correlation function from which the flow was extracted. In [171], all gauge couplings were extracted from the fermion-fermion-gauge vertex, and the

Newton coupling from the three-graviton vertex, at the momentum-symmetric point. For more details, we refer to [171].

> (97) 中的耦合根据提取流所用的关联函数不同具有不同形式。文献 [171] 中，所有规范耦合都从动量对称点的费米子-费米子-规范顶点提取，牛顿耦合从三引力子顶点提取。更多细节参见文献 [171]。

We proceed with the discussion of several points that need to be handled with care in order to obtain full SM flows if aiming for quantitative precision required for even a qualitative resolution of the phase structure of the ASSM:

> 若要得到完整的标准模型流，实现解析 ASSM 相结构 (哪怕是定性解析) 所需的定量精度，我们接下来讨论几个需要谨慎处理的问题:

Strong IR QCD sector In the IR regime for $k, p \lesssim 10\mathrm{GeV}$, one needs an improved approximation for the gluonic sector for including confinement, as well as in the matter sector for including dynamical spontaneous chiral symmetry breaking; see [235]. Therefore, results from functional approaches for 2 and 2+1 flavor computations were utilized which match lattice 2 and 2+1 flavor benchmarks in the IR regime of physical QCD [235-237]. With this external input, the computation is able to effectively describe the mass generation of quarks due to chiral symmetry breaking.

> 强红外 QCD 区对于 $k, p \lesssim 10\mathrm{GeV}$ 的红外区域，胶子区需要包含禁闭的改进近似，同样物质区也需要包含动力学自发手征对称性破缺的改进近似，参见文献 [235]。因此，本文采用了 2 味和 2+1 味泛函方法的计算结果，这些结果在物理 QCD 的红外区域符合格点 2 味和 2+1 味基准结果 [235-237]。借助这一外部输入，该计算可以有效描述手征对称性破缺导致的夸克质量生成。

Matching of the top pole mass For most Yukawa couplings, it is sufficient to fix their values in the IR with the Euclidean curvature mass, e.g., with the simple relation $y_{b,k=0} = \sqrt{2}M_{b,\,\mathrm{pole}}/v$ for the bottom quark. For the top quark, the situation is different since the SM flows are very sensitive to the top mass. In [171], the nontrivial relation between the pole and the Euclidean curvature mass of the top quark was taken into account by determining the pole in the complex momentum plane of the two-point function for a given Yukawa coupling. This leads to the relation

> 顶夸克极点质量匹配对于大多数汤川耦合，在红外区用欧几里得曲率质量固定其数值就足够了，例如底夸克可以通过简单关系 $y_{b,k=0} = \sqrt{2}M_{b,\,\mathrm{pole}}/v$ 完成固定。而顶夸克的情况不同，因为标准模型演化对顶夸克质量非常敏感。文献 [171] 中，通过对给定汤川耦合确定两点函数在复动量平面上的极点，计入了顶夸克极点质量与欧几里得曲率质量之间的非平凡关系，得到关系如下

$$m_t = 165.4^{+0.9}_{-0.2}\mathrm{GeV} \;\leftrightarrow\; y_t = 0.950^{+0.005}_{-0.001}. \tag{98}$$

Determining the pole in the complex plane additionally provides a prediction for the decay width of the top quark, which is in agreement with experiments [171].

> 在复平面上确定极点还可以给出顶夸克衰变宽度的预言，该预言与实验结果一致 [171]。

Strong UV gravity sector Since the strong UV gravity sector has several important points, we will address that in the next subsection. At first, we will discuss the impact of matter fluctuations on the UV fixed point, which is key for the existence of a combined SM-gravity fixed point. Then we will look at the impact of gravity fluctuations on the matter sector. The matter sector in the SM contains UV Landau poles at high-energy scales which need to be avoided by gravity fluctuations.

强紫外引力区由于强紫外引力区包含多个要点，我们将在下一小节展开讨论。首先，我们将讨论物质涨落对紫外固定点的影响，这是标准模型-引力联合固定点存在的关键。随后我们会探讨引力涨落对物质区的影响。标准模型的物质区在高能量标度下存在紫外朗道极点，需要通过引力涨落消除该极点。

Gravity-Matter Interplay

引力-物质相互作用

We initiate our discussion of the phase structure of the ASSM with that of the impact of the matter fluctuations on the fixed point of the gravity couplings (87). For simplicity, we consider at first only minimally coupled matter fields: matter fields where the coupling to gravity only comes via the kinetic term and no matter self-interactions are considered. Starting from the pure gravity fixed point (87), one adds successively scalars, fermions, and gauge bosons. The current state-of-the-art computations can be found in [120] (scalars), [197] (fermions), and [196] (gauge bosons), where the momentum-dependent running of all two-point functions as well as the running of the three-graviton vertex and the graviton-matter vertex was taken into account. Therefore, these systems contain several momentum-dependent anomalous dimensions: $\eta_h\left(p^2\right), \eta_c\left(p^2\right), \eta_{\text{mat}}\left(p^2\right)$, two avatars of the cosmological constant, the graviton mass parameter μ and the avatar from the three graviton vertex λ_3, as well as two avatars of the Newton coupling, g_3 from the three-graviton vertex and g_{mat} from the matter-graviton vertex.

我们从物质涨落对引力耦合常数不动点的影响开始讨论渐近安全标准模型 (ASSM) 的相结构 (87)。为简化起见，我们首先仅考虑最小耦合物质场: 这类物质场与引力的耦合仅来自动能项，且不考虑物质自相互作用。从纯引力不动点 (87) 出发，我们依次引入标量场、费米子和规范玻色子。当前最先进的计算可见文献 [120](标量场)、[197](费米子) 和 [196](规范玻色子)，这些工作同时考虑了所有两点函数依赖动量的跑动、三引力子顶点的跑动以及引力子-物质顶点的跑动。因此，这些系统包含多个依赖动量的反常量纲: $\eta_h\left(p^2\right), \eta_c\left(p^2\right), \eta_{\text{mat}}\left(p^2\right)$，宇宙学常数的两种形式，引力子质量参数 μ 与三引力子顶点的形式 λ_3，以及牛顿耦合的两种形式，来自三引力子顶点的 g_3 和来自物质-引力子顶点的 g_{mat} 。

The results are displayed in Fig. 7 and consolidate the first fluctuation work in this direction [198]. The left panel of Fig. 7 shows the fixed-point values as a function of the number of scalar fields N_s. The Newton couplings diverge around $N_s \approx 52$, which is in a regime where the truncation cannot be trusted anymore. In [89], it was suggested that an expansion about a background that is a solution to the quantum equation of motion might remove the divergence in the scalar case.

结果展示在图 7 中，巩固了该方向上的第一项涨落研究工作 [198]。图 7 左面板给出了不动点值关于标量场数目 N_s 的函数关系。牛顿耦合在 $N_s \approx 52$ 附近发散，该区域处于截断不再可靠的范围。文献 [89] 提出，围绕满足量子运动方程的背景做展开或许可以消除标量场情形下的发散。

The results for minimally coupled fermions are displayed in the central panel in Fig. 7. The Newton couplings decrease with the number of fermionic flavors, and arbitrarily many fermions can be included in the system.

最小耦合费米子的结果展示在图 7 的中间面板。牛顿耦合随费米子味数目增加而减小，该系统可容纳任意多的费米子。

In the gauge-field case, the fixed point is disappearing in the complex plane due to a fixed-point merger at $N_v \approx 13$. In [196] it was shown that this strongly depends on the regulator: with a slightly different regulator, arbitrarily many gauge bosons can be included in the system. This indicates that the merger is unphysical and an artifact of the truncation.

在规范场情形，不动点会因 $N_v \approx 13$ 处发生不动点合并而消失在复平面。文献 [196] 表明，这一结果强烈依赖于调节器: 使用略有不同的调节器时，该系统可以容纳任意多的规范玻色子。这说明该合并是非物理的，是截断带来的人工产物。

In all panels of Fig. 7, we can observe that the Newton coupling from the three-graviton vertex has similar values and behaves similarly to the Newton coupling from the graviton-matter vertices. This effect was dubbed effective universality [91, 120] as already discussed in section "Fixed Point and Critical Exponents." In [91], it was shown that this effect also extends to the ghost-graviton vertex and is only present in a small parameter space of the couplings, which is precisely where the UV fixed point is located. While the different Newton couplings g_n are related by STIs (see section "Background Independence and Symmetry Identities") it is not expected that their behavior is that similar in the presence of further nontrivial operators beyond the curvature scalar. In fact, we would expect large differences between the couplings in a highly non-perturbative setting as the g_n ' s also have overlap with higher-curvature terms as well as terms with curvature and covariant momentum dependences; for pure gravity, this was discussed in [114]. Therefore, the existence of effective universality was taken as evidence of a close-perturbative nature of the UV fixed point [91] with the relevant operators R, Λ and R^2 . In the right panel of Fig. 7, we see that effective universality is increasingly violated with an increasing number of gauge bosons, and the system runs into a fixed-point merger in this approximation. This is not the case if different regulators are chosen; see [196]. Moreover, it was shown there that the gauge boson-gravity systems in the minimally coupled approximation always have a Reuter-type fixed point as they can be mapped onto a pure gravity system with effective Newton constant and cosmological constant. In conclusion, the fixed-point merger signals a failure of the approximation to the flow in this minimally coupled system and not a disappearance of the fixed point. Different regulators or shifted relative cutoff scales are required to stabilize the approximation. Only then do the flows show the above feature of the system that it has a Reuter-type fixed point.

在图 7 的所有面板中，我们可以观察到，三引力子顶点给出的牛顿耦合与引力子-物质顶点给出的牛顿耦合数值相近，行为也相似。正如“不动点与临界指数”一节已经讨论过的，该效应被称为有效普适性 [91, 120]。文献 [91] 指出，该效应同样延伸至鬼-引力子顶点，且仅存在于 couplings 的一个小参数空间中，而紫外不动点正好就位于这个区域。尽管不同的牛顿耦合 g_n 满足斯莱文诺夫-泰勒恒等式 (见“背景独立性与对称性恒等式”一节)，但在曲率标量之外还存在其他非平凡算符的情况下，我们并不预期它们的行为会如此相似。事实上，我们预期这些耦合在高度非微扰的情况下会存在很大差异，因为 g_n 也会与高曲率项，以及带有曲率和协变动量依赖的项存在重叠；纯引力下的相关讨论可见文献 [114]。因此，有效普适性的存在被视作紫外不动点具有近微扰性质的证据 [91]，相关算符为 R, Λ 和 R^2 。在图 7 的右面板中我们可以看到，有效普适性随着规范玻色子数增加被愈发严重地破坏，在该近似下系统进入不动点合并。如果选取不同的调节器则不会出现这种情况，见文献 [196]。此外该文献还指出，最小耦合近似下的规范玻色子-引力系统始终存在罗伊特型不动点，因为这类系统可以被映射为带有有效牛顿常数和宇宙学常数的纯引力系统。综上，不动点合并标志着该最小耦合系统中流近似失效，而非不动点消失。需要不同的调节器或偏移的相对截断标度来稳定近似，只有这样流才能体现出系统存在罗伊特型不动点这一特征。

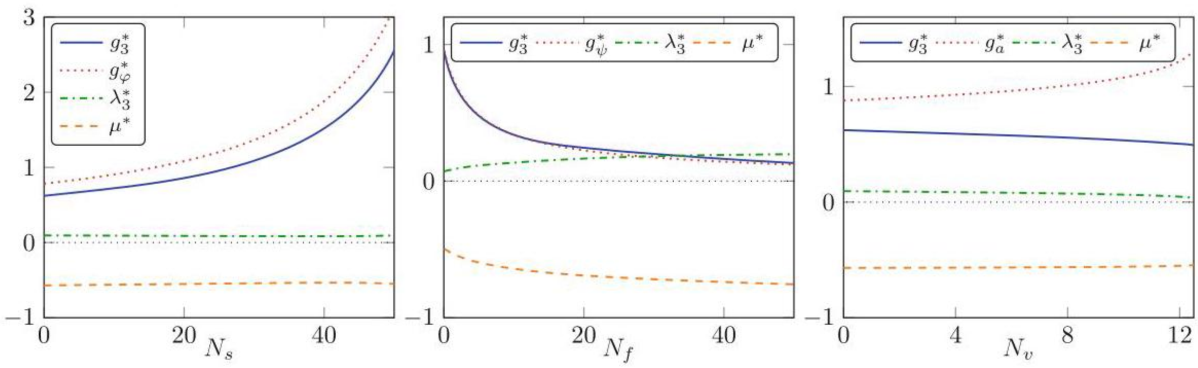


Fig. 7 Fixed-point values of the fluctuation couplings as a function of the number of scalar (left), fermion (middle), and gauge fields (right). All truncations include the graviton two- and three-point function as well as the respective graviton-matter vertex. In the scalar case, the Newton couplings, g_3 and g_φ , are diverging at $N_s \approx 52$. The fermionic case is stable for all N_f . In the gauge field case, the fixed point disappears in the complex plane at $N_v \approx 13$. It was explained in [196] that the vanishing of the fixed point is an artifact of the truncation and how it can be lifted in the gauge-field case. In [89], it was suggested that an expansion about a background that is a solution to the quantum equation of motions might remove the divergence in the scalar case. (The result are taken from [120] (scalar), [197] (fermion), and [196] (gauge))

图 7 涨落耦合的不动点值随标量 (左)、费米子 (中) 和规范场 (右) 数目的变化。所有截断都包含引力子两点函数、三点函数，以及对应的引力子-物质顶点。标量情形下，牛顿耦合 g_3 和 g_φ 在 $N_s \approx 52$ 处发散。费米子情形对所有 N_f 都稳定。规范场情形下，不动点在 $N_v \approx 13$ 处进入复平面消失。文献 [196] 已经说明，不动点消失是截断带来的人为结果，并解释了在规范场情形下该问题如何解决。文献 [89] 提出，在满足量子运动方程的背景上做展开或许可以消除标量情形下的发散。(结果分别取自 [120](标量)、[197](费米子) 和 [196](规范场))

In [171], the problem was circumvented with the observation that SM matter content can be easily accommodated with the approximation of effective universality, $g_3 = g_a = g_\varphi = g_\psi = g_c$, leading to the SM fixed point:

文献 [171] 中，该问题通过一个观察得到规避: 标准模型物质内容可以很容易地被纳入有效普适性近似 $g_3 = g_a = g_\varphi = g_\psi = g_c$ ，最终得到标准模型不动点:

$$(g_3, \lambda_3, \mu)^*_{\mathrm{SM}} = (0.17, 0.15, -0.71)\,, \tag{99}$$

with the critical exponents

临界指数为

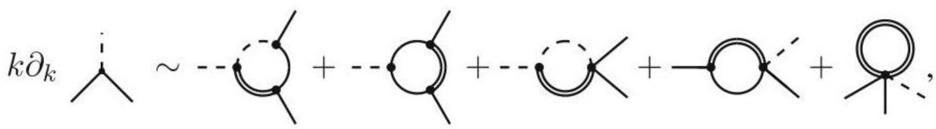
$$\theta_i^{\mathrm{SM}} = (-2.9 \pm 4.5\mathrm{i}, 10.2)\,. \tag{100}$$

We expect that the fixed-point values will be similar in an improved truncation that does not possess the fixed-point merger in N_v due to the effective universality of the Newton couplings. Consequently, these values were utilized in the computation in [171].

我们预期，由于牛顿耦合的有效普适性，在不会导致 N_v 出现不动点合并的改进截断中，不动点值仍然是相近的。因此文献 [171] 的计算中就使用了这些数值。

Next, we discuss the impact of graviton fluctuation on the marginal SM matter couplings. For example, the graviton contribution to the Yukawa coupling is generated by the diagrams:

接下来，我们讨论引力子涨落对边缘标准模型物质耦合的影响。例如，汤川耦合的引力子贡献由以下费曼图产生:



(101)

where double lines represent gravitons, dashed lines scalars, and solid lines fermions. We have suppressed the prefactors of the diagrams and the regulator insertions for simplicity. All diagrams in (101) are linear in the Yukawa coupling, but higher-order contributions of the Yukawa coupling enter through the fRG resummation of higher-order interactions. It has been common practice to use the parametrization [225, 238-242]:

其中双线代表引力子，虚线代表标量，实线代表费米子。为简化起见，我们省略了图的 prefactor 和调节器插入。式 (101) 中所有图都对汤川耦合是线性的，但汤川耦合的高阶贡献会通过泛函重整化群对高阶相互作用的重求和进入。通常的参数化写法是 [225, 238-242]:

$$\partial_t y|_{\mathrm{gravity}} = -f_y\,(G_{\mathrm{N}}, \mu, \ldots)\, y, \tag{102}$$

where f_y only depends on the gravity couplings. In the fixed-point regime beyond the Planck scale, f_y is in a good approximation a constant, while it vanishes below the Planck scale. Many applications used f_y

as a constant-free parameter beyond the Planck scale for the fixed-point analysis of (beyond) the SM physics and then utilized standard perturbative beta functions which vanish f_y below the Planck scale [243-254]. The gravity contributions to the gauge and scalar quartic couplings are commonly parametrized in straight analogy:

其中 f_y 仅依赖于引力耦合。在普朗克尺度以上的不动点区域，f_y 近似为常数，在普朗克尺度以下则趋于零。诸多研究在 (超出) 标准模型物理的不动点分析中，将 f_y 作为普朗克尺度以上的无常数参数，随后使用标准微扰 β 函数，这类 β 函数在普朗克尺度以下令 f_y 消失 [243-254]。引力对规范耦合和标量四次耦合的贡献通常按照如下直接类比参数化:

$$\partial_t g = -f_g g,\ \partial_t \lambda = -f_\lambda \lambda. \tag{103}$$

The signs and the size of the gravity contributions to the marginal SM couplings are crucial for the compatibility of the SM with asymptotically safe gravity. They were analyzed in with fRG techniques in [196, 213, 214, 221, 225, 255-257] (gauge coupling), [210, 217, 218, 238, 258] (Yukawa coupling), and [208-210, 226, 240, 258-260] (quartic scalar coupling), as well as with perturbative techniques [261-269].

引力对标准模型边缘耦合贡献的符号和大小，是标准模型与渐近安全引力兼容的关键。已有研究利用泛函重整化群 (fRG) 技术分别对其做了分析: 规范耦合见 [196, 213, 214, 221, 225, 255-257]，汤川耦合见 [210, 217, 218, 238, 258]，标量四次耦合见 [208-210, 226, 240, 258-260]，此外也有微扰技术的相关研究 [261-269]。

For the compatibility of the U (1) hypercharge coupling with asymptotic safety, $f_g \geq 9.8 \cdot 10^{-3}$ is needed [225]. The Yukawa sector is more intricate and also depends on whether the hypercharge coupling becomes asymptotically safe or free. Assuming an asymptotically free hypercharge coupling, $f_y \geq 10^{-4}$ is needed [239, 245]. The quartic scalar coupling will be discussed below.

若要让 U (1) 超荷耦合与渐近安全兼容，需要满足 $f_g \geq 9.8 \cdot 10^{-3}$ [225]。汤川区更为复杂，其结果还依赖于超荷耦合是渐近安全还是渐近自由。假设超荷耦合是渐近自由的，则需要满足 $f_y \geq 10^{-4}$ [239, 245]。标量四次耦合将在下文讨论。

In [171], the regulator and momentum dependence of the gravity contributions to the gauge and Yukawa coupling were investigated. The regulator dependence was assessed by integrating out gravity and matter fluctuations at different scales, k_{grav} and k_{mat}. The parameter $\gamma_{\text{mg}} = k_{\text{mat}}/k_{\text{grav}}$ describes the ratio of these cutoff scales; see [171] for more details. A stable truncation should be insensitive to a change of cutoff scales of order one.

文献 [171] 研究了引力对规范和汤川耦合贡献的正规化子与动量依赖性。通过在不同尺度 k_{grav} 和 k_{mat} 积掉引力和物质涨落，对正规化子依赖性做了评估。参数 $\gamma_{\text{mg}} = k_{\text{mat}}/k_{\text{grav}}$ 描述了这些截断尺度的比值，更多细节参见 [171]。稳定的截断应当对一阶的截断尺度变化不敏感。

The results of [171] are displayed in Fig. 8. The left panel shows the gravity contribution to the gauge coupling, which was computed from the quark-gluon vertex. At vanishing momentum, the contribution has a definite sign which is compatible with asymptotic safety supporting asymptotic freedom. For $\gamma_{\text{mg}} = 0$, the contribution exactly vanishes due to a kinematic identity that was also found in other gauge correlation functions [196,213].

文献 [171] 的结果展示在图 8 中。左图为引力对规范耦合的贡献，由夸克-胶子顶点计算得到。动量为零时，该贡献符号确定，与支持渐近自由的渐近安全相容。对于 $\gamma_{\mathrm{mg}} = 0$，该贡献因运动学恒等式恰好为零，这一恒等式在其他规范关联函数中也有发现 [196,213]。

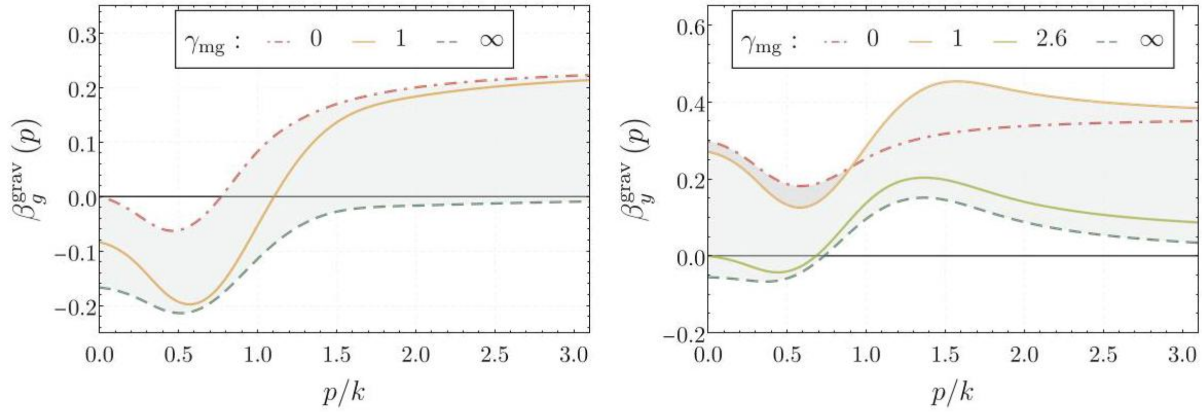


Fig. 8 Matter-gravity contributions to the gauge beta functions β_g^{grav} derived from the gauge-fermion vertex for $g = 1, g_h = 1, \mu_h = 0$, and different values of γ_{mg}. For $\gamma_{\mathrm{mg}} = 0, \beta_g^{\mathrm{grav}}$ vanishes due to an exact cancellation between the fermion anomalous dimension and the three-point diagrams. (Taken from [171])

图 8 物质-引力对规范 β 函数 β_g^{grav} 的贡献，由规范-费米子顶点在 $g = 1, g_h = 1, \mu_h = 0$ 下导出，对应 γ_{mg} 的不同取值。对于 $\gamma_{\mathrm{mg}} = 0, \beta_g^{\mathrm{grav}}$，该贡献因费米子反常维数与三点图之间的精确抵消而为零。(引自 [171])

The situation is qualitatively different in the Yukawa sector as shown in the right panel of Fig. 8. Two different signs are possible at vanishing momentum, and the crossover happens at $\gamma_{\mathrm{mg}} = 2.6$ which is an order one change in the cutoff scales. For the naive choice of cutoff scales, $\gamma_{\mathrm{mg}} = 1$, the gravity contributions drive the Yukawa coupling into a Landau pole, and only for $\gamma_{\mathrm{mg}} > 2.6$, the couplings become asymptotically safe or free. This computation emphasizes the urgency to improve computations in the Yukawa sector. We note large negative values of the cosmological constant can stabilize the computation [239], which is not realized in the SM fixed point (99), or the inclusion of ultralocal higher-derivative interactions [238], i.e., working with the classical Stelle action (6).

如图 8 右图所示，汤川区的情况在定性上完全不同。动量为零时贡献可以有两种不同符号，突变发生在 $\gamma_{\mathrm{mg}} = 2.6$ 处，对应截断尺度的一阶变化。若采用截断尺度的朴素选择 $\gamma_{\mathrm{mg}} = 1$，引力贡献会将汤川耦合推到朗道极点，仅当满足 $\gamma_{\mathrm{mg}} > 2.6$ 时，耦合才会变成渐近安全或渐近自由。该计算凸显了改进汤川区计算的紧迫性。我们注意到，大的负宇宙常数可以稳定计算 [239]，但这在标准模型不动点 (99) 中无法实现；或者也可以引入超局域高阶导数相互作用 [238]，即采用经典斯泰勒作用量 (6) 来计算。

In case of the quartic scalar coupling, the sign of f_λ was found to be negative [208-210,226,240,258-260] leading to an irrelevant direction at the $\lambda = 0$ fixed point. The irrelevant direction can be used to predict the ratio between Higgs and top mass, which led to the result $m_H = 125\mathrm{GeV}$ with one-loop beta functions [270]. The result shifts to $m_H = 132\mathrm{GeV}$ with higher-order beta functions and a more accurate determination of the top-quark pole mass; see [171] and also below. Note however that this result is very sensitive to the value of the top mass, just as the metastability of the Higgs potential in perturbative computations [271-273]. Furthermore, the value is modified beyond SM physics [244, 246, 248, 251, 274].

对于四次标量耦合，已发现 f_λ 的符号为负 [208-210,226,240,258-260]，这会在 $\lambda = 0$ 不动点处产生一个无关方向。利用该无关方向可以预言希格斯质量与顶夸克质量的比值，在单圈 beta 函数下得到结果 $m_H = 125\text{GeV}$ [270]。若采用高阶 beta 函数并对顶夸克极点质量做更精确的测定，结果会移动到 $m_H = 132\text{GeV}$；参见文献 [171] 以及下文。但请注意，该结果对顶夸克质量的数值非常敏感，这和微扰计算中希格斯势的亚稳定性情况类似 [271-273]。此外，超出标准模型的新物理也会改变该数值 [244, 246, 248, 251, 274]。

A natural question to ask is if there are other fixed points in the scalar sector that can connect to the SM in the IR, in particular due to the small discrepancy between the measured value of the Higgs mass and the prediction by asymptotic safety for $\lambda^* = 0$. In [171], the Higgs fixed-point potential was investigated in a high-order Taylor expansion. A fixed point was identified that converges to the Gaußian fixed point but contains a second relevant direction. For more details, we refer to [171], and for more details on gravity-matter interactions, we refer to [17].

一个很自然的问题是，标量 sector 中是否还存在其他能在红外区连接到标准模型的不动点，尤其是因为测得的希格斯质量数值和渐近安全对 $\lambda^* = 0$ 的预言之间存在微小偏差。文献 [171] 中通过高阶泰勒展开研究了希格斯不动点势，找到了一个收敛到高斯不动点但包含第二个相关方向的不动点。更多细节参见文献 [171]，关于引力-物质相互作用的更多细节参见文献 [17]。

Phase Structure of the Asymptotically Safe Standard Model

渐近安全标准模型的相结构

With the approximations and details discussed in the last section, we are now ready to look at full UV-IR flows for all SM couplings. The result from [171] is shown in Fig. 9. All marginal SM couplings emerge from the Gaußian fixed point in the UV and reach their physical values in the IR. The quartic Higgs coupling approaches the Gaußian fixed point from below and turns negative around $k = 10^{10}\text{GeV}$, which is compatible with the metastability scale found in perturbative studies. Threshold effects in the IR and in the UV have been included which causes most couplings to freeze out below the electroweak symmetry breaking scale, which is around 1TeV in the RG-scale k .

借助上一节讨论的近似与细节, 我们现在可以分析所有标准模型耦合的完整紫外-红外流。文献 [171] 的结果如图 9 所示。所有边缘标准模型耦合都从紫外的高斯不动点出发，在红外达到其物理值。希格斯四次耦合从下方趋近高斯不动点，在约 $k = 10^{10}\text{GeV}$ 处变为负值，这与微扰研究中得到的亚稳标度一致。本文已包含红外和紫外的阈值效应，因此大多数耦合在电弱对称破缺标度 (在重整化群标度 k 下约为 1TeV) 以下冻结。

Not all values of the Higgs and top mass can be connected to the UV fixed point, which is displayed in Fig. 10. The green area can be connected to the nontrivial

并非所有希格斯质量和顶夸克质量都能连接到紫外不动点，如图 10 所示。绿色区域可以连接到非平凡

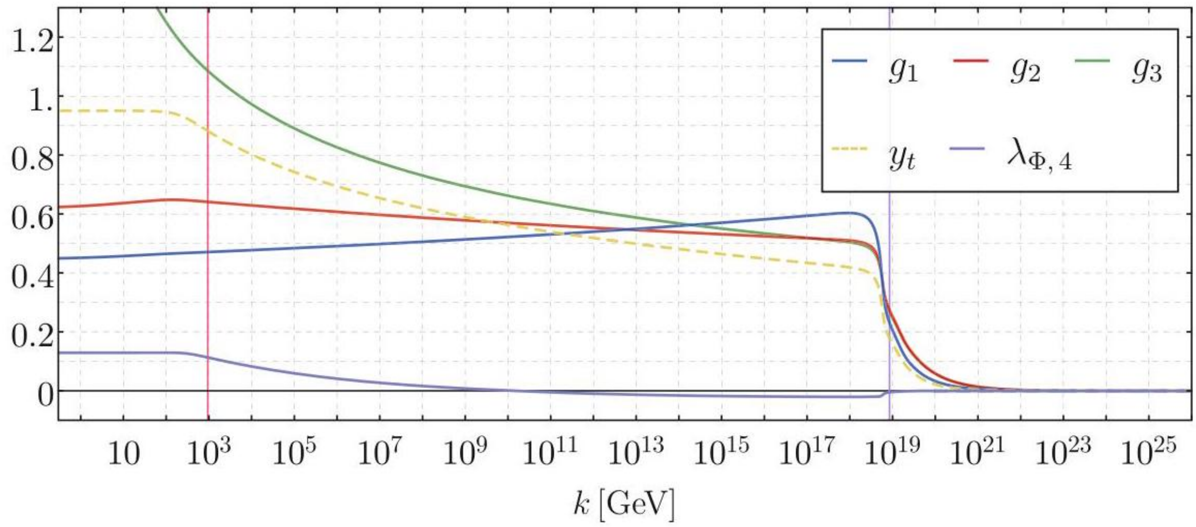
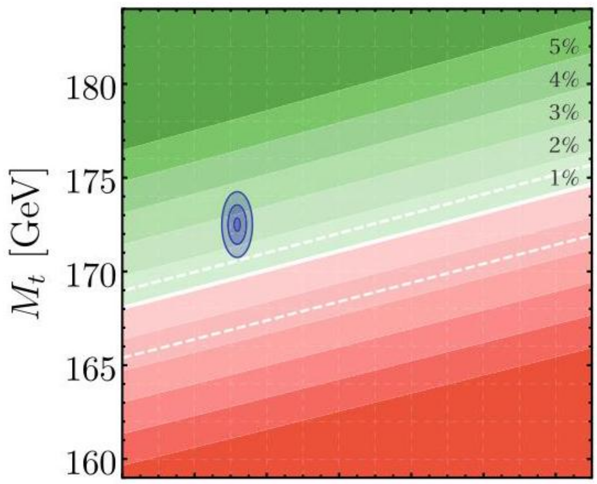


Fig. 9 We show the RG running of the classically marginal SM couplings including quantum gravity and threshold effects. The threshold effects due to the masses of the particle cause couplings to freeze out in the IR, with the exception of the strong and the electromagnetic coupling. In the UV, quantum gravity drives all couplings into the Gaußian fixed point. (Taken from [171])

图 9 我们展示了包含量子引力和阈值效应的经典边缘标准模型耦合的重整化群跑动。粒子质量带来的阈值效应使耦合在红外冻结，强耦合和电磁耦合除外。在紫外，量子引力将所有耦合驱动至高斯不动点。(取自 [171])



122 124 126 128 130 132 134

M_H [GeV]

Fig. 10 IR masses of the Higgs boson and top quark that can be reached from the asymptotically safe UV. The green region can be approached from the nontrivial Gaußian fixed point with two relevant directions, the white line can be approached from the standard Gaußian fixed point with one relevant direction, and the red region does not have a UV completion. The blue circles represent the central measured values including the $1\sigma, 3\sigma$, and 5σ uncertainty. (Taken from [171])

图 10 可从渐近安全紫外区到达的希格斯玻色子和顶夸克的红外质量。绿色区域可从具有两个相关方向的非平凡高斯不动点趋近，白色线可从具有一个相关方向的标准高斯不动点趋近，红色区域不存在紫外完备性。蓝色圆点代表包含 $1\sigma, 3\sigma$ 和 5σ 不确定度的中心测量值。(取自 [171])

Gaußian fixed point. The area is bounded by the white line, which is precisely the line that connects to the predictive standard Gaußian fixed point. On the other side of the white line, we have the red regime for which no UV completion was found.

高斯不动点。该区域由白色线界定，这条白线恰好是连接到可预言性标准高斯不动点的线。白线的另一侧是红色区域，未找到该区域的紫外完备性。

The experimental values are shown in Fig. 10 with blue circles including the $1\sigma, 3\sigma$, and 5σ uncertainty. They lay in the green regime with roughly a 2.9GeV distance between the central values and the white Gaußian line. The combined 5σ experimental and theoretical error are roughly of the same size implying that it is not excluded but unlikely that the pure SM lays in the red regime or on the white line. This can certainly change upon inclusion of beyond SM physics such as dark matter or right-handed neutrinos. For example, the asymptotically safe dark matter models in [244,248,251] allow for a correct Higgs mass from the standard Gaußian UV fixed point.

实验值如图 10 中的蓝色圆点所示，包含 $1\sigma, 3\sigma$ 和 5σ 不确定度。实验值位于绿色区域，中心值与白色高斯线之间的距离约为 2.9GeV 。结合后的 5σ 实验与理论误差大小大致相当，这意味着纯标准模型处于红色区域或白线上的结论并未被排除，但可能性较低。如果纳入超出标准模型的新物理 (例如暗物质或右手中微子)，结论肯定会发生变化。例如， 文献 [244,248,251] 中的渐近安全暗物质模型允许标准高斯紫外不动点给出正确的希格斯质量。

Outlook

展望

The results on the phase structure of the asymptotically safe Standard Model reviewed in this section constitute an important milestone in the quest for reliable predictions in asymptotically safe particle physics. It is the first self-consistent computation: all running coupling parameters are obtained within the same RG framework, which is of paramount importance for reliable predictions and the systematic improvement within the given vertex expansion scheme. It is based on momentum-dependent correlation functions and hence allows the computation of on-shell observables such as pole masses, in contradistinction to other approaches. In summary, these results and their systematics in the fluctuation approach open a path toward reliable computations with increasingly small systematic errors and hence usher in the era of quantitative predictivity in asymptotically safe particle physics.

本节所综述的渐近安全标准模型相结构研究结果，是探寻渐近安全粒子物理可靠预言过程中的重要里程碑。这是首次自洽计算：所有跑动耦合参数都在同一个重整化群框架下得到，这对获得可靠预言以及在给定顶点展开方案内进行系统性改进至关重要。本计算基于动量依赖关联函数，因此能够计算极点质量这类在壳可观测量，这是与其他方法的不同之处。总而言之，这些结果及其在涨落方法中的系统性为系统误差不断降低的可靠计算开辟了道路，也由此开启了渐近安全粒子物理的定量预言时代。

Lorentzian Quantum Gravity

洛伦兹量子引力

In this final section, we discuss a direct fRG approach to Lorentzian metric quantum gravity. It allows what can be considered the holy grail of any approach to quantum gravity: direct non-perturbative computations within a Lorentzian signature. Such an approach has been set up the first time in [82]. The spectral fRG approach underlying [82] has been discussed in detail in [83] and is based on the spectral functional approach put forward in [81].

在最后这一节中，我们讨论面向洛伦兹度规量子引力的直接泛函重整化群 (fRG) 方法。该方法能够实现所有量子引力研究都追求的终极目标：在洛伦兹号差下进行直接非微扰计算。这一方法最早在文献 [82] 中被提出。文献 [82] 所基于的谱 fRG 方法已在文献 [83] 中得到详细讨论，该方法建立在文献 [81] 提出的谱泛函方法的基础上。

Fundamentals and Effective Field Theory

基础与有效场论

We start this section with a brief overview of the state of the art concerning timelike correlation functions in asymptotically safe gravity or more generally Lorentzian quantum gravity. To date, most computations in asymptotically safe quantum gravity including those in the fluctuation approach reviewed so far have been performed with a Euclidean signature. The Euclidean signature allows for a simple ordering principle of the momentum fluctuations and removed poles and cuts from the integration domain. This property is lost with Lorentzian signature since the squared momentum can vanish, $p^2 = 0$, while, for example, the spatial momentum is large. The ordering of momentum fluctuations is however important for the coarse gaining procedure of the fRG. Moreover, for numerical integrations of loop momenta, the absence of singularities in or close to the integration contour is essential.

我们在本节开篇简要概述类时关联函数在渐近安全引力 (更宽泛地说，洛伦兹量子引力) 中的研究现状。到目前为止，渐近安全量子引力的绝大多数计算，包括我们迄今回顾的涨落方法相关计算，都是在欧几里得符号下完成的。欧几里得符号为动量涨落提供了简单的排序规则，还能从积分区域中消去极点和割线。洛伦兹符号不具备这一性质，因为平方动量可以为零，$p^2 = 0$，例如当空间动量很大时就是如此。而动量涨落的排序对函数重整化群 (fRG) 粗粒化过程至关重要。此外，对于圈动量的数值积分，积分围道内或附近不存在奇点是必不可少的条件。

In principle, it is feasible to first perform the integrating out of all momentum fluctuations and then Wick rotate the results to Lorentzian signature; see [92] for an explicit example. However, the Wick rotation is already a subtle issue on flat Minkowski space-times, and it is further complicated by the dynamical nature of the metric quantum gravity. Moreover, the Wick rotation based on numerical data comes with an ill-conditioned or even ill-posed reconstruction problem, and the systematic error of the results has to be evaluated with great care [92]. For general considerations on the reconstruction problem, see, e.g., [90, 275-279].

原则上，我们可以先积去所有动量涨落，再将结果威克转动到洛伦兹符号；具体例子可见文献 [92]。但威克转动在平直闵氏时空中本身就是一个微妙的问题，在量子引力中，由于度规的动力学性质，问题会进一步复杂化。此外，基于数值数据做威克转动会带来病态甚至不适定的重构问题，因此必须非常谨慎地评估结果的系统误差 [92]。关于重构问题的一般性讨论，参见例如 [90, 275-279]。

For all of the above reasons, approaches that allow for direct non-perturbative computations with Lorentzian signature are much wanted. First steps toward computations with Lorentzian signature have been reported in [58, 280-290]; for attempts in other approaches to quantum gravity, see, e.g., [48, 291-297].

基于上述所有原因，能够直接对洛伦兹符号做非微扰计算的研究方法是目前迫切需要的。走向洛伦兹符号计算的初步工作已在 [58, 280-290] 中报道；其他量子引力研究方向的相关尝试参见例如 [48, 291-297]。

Recently, several milestones have been achieved in the fluctuation approach: In [92], the graviton propagator was reconstructed on the basis of numerical data for the Euclidean or spacelike propagator, resulting in a graviton spectral function and hence a graviton propagator for spacelike and timelike momenta. In [82], the first direct computation of asymptotically safe gravity in Lorentzian background was put forward. This allowed us to compute the graviton spectral function and hence the space- and timelike graviton propagator. The results of this direct computation corroborated that of the reconstruction procedure in [92]. Moreover, it is worth emphasizing that the spectral approach to asymptotically safe gravity gains much from recent results on spectral properties of QCD and Yang-Mills theory. In some sense, spectral considerations in these theories face even more intricacies as in asymptotically safe gravity: while the latter enjoys effective universality and a close-perturbative behavior also signaled by a relatively weak gravitational coupling even in the UV scaling regime, the former has a truly non-perturbative IR regime with emergent resonances and confinement, which manifests as a dynamical mass gap in the gluon propagator. For respective investigations in QCD, see [81, 95-97]; for the discussion of the conformal regime of QCD close to the Banks-Zaks fixed point highly relevant in the present context, see [298].

近年来，涨落方法领域已经取得了多个里程碑进展: 在文献 [92] 中，作者基于欧几里得或类空传播子的数值数据重构了引力子传播子，得到了引力子谱函数，进而得到了同时适用于类空和类时动量的引力子传播子。在文献 [82] 中，研究者首次在洛伦兹背景下完成了渐近安全引力的直接计算。这让我们得以计算引力子谱函数，进而得到类空和类时的引力子传播子。该直接计算的结果证实了文献 [92] 中重构流程得到的结论。此外，值得强调的是，渐近安全引力的谱方法很大程度上受益于 QCD(量子色动力学) 和杨-米尔斯理论谱性质的最新研究成果。从某种意义上说，这些理论中的谱问题比渐近安全引力更为复杂: 后者既具有有效普适性，还在紫外标度区表现出近微扰行为，引力耦合也相对较弱；而前者存在真正的非微扰红外区，伴随着涌现共振和禁闭，这在胶子传播子中表现为动力学质量隙。QCD 中的相关研究参见 [81, 95-97]；与本文背景高度相关、接近班克斯-扎克斯不动点的 QCD 共形区的讨论，参见 [298]。

The spectral functional approach is based on the spectral representation of correlation functions, and most prominently on that of the propagator, the Källén-Lehmann (KL) spectral representation [299, 300] as already discussed in section "Lorentzian Flow Equation for the Effective Action" around (30). The KL spectral representation is a powerful tool which allows the encoding of the propagator in the entire complex momentum plane in just one real function. It provides intuitive access to physics properties such as on-shell states and unitarity properties. If the KL representation of the propagator of a given field exists, the spectral function and the propagator are related via

谱函数方法基于关联函数的谱表示，其中最核心的是传播子的卡伦-莱曼 (KL) 谱表示 [299, 300]，我们已经在“有效作用量的洛伦兹流方程”一节围绕式 (30) 做过讨论。KL 谱表示是一个强大的工具，它仅用一个实函数就能编码整个复动量平面上传播子的全部信息。它还能让我们更直观地研究物理性质，比如在壳态和幺正性。如果给定场的传播子存在 KL 表示，那么谱函数和传播子就满足以下关系

$$G\left(p_0, |\mathbf{p}|\right) = \int_{0_-}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \frac{\lambda \rho\left(\lambda, |\mathbf{p}|\right)}{\lambda^2 + p_0^2} = \int_{0_-}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \frac{\lambda \rho\left(\lambda\right)}{\lambda^2 + p^2}, \; \rho\left(\lambda\right) = \rho\left(\lambda, 0\right),$$

(104)with the temporal and spatial momentum p_0 and $\mathbf{p}$ and the spectral values λ . For the sake of simplicity, we again restricted ourselves to the scalar case. The lower bound of the spectral integral, 0_- , takes into account that for massless particles, the spectral function contains a delta function at vanishing spectral values, $\delta\left(\lambda\right)$.

其中时间和空间动量分别为 p_0 和 $\mathbf{p}$ ，谱值为 λ 。为简单起见，我们再次将讨论限制在标量情形。谱积分的下界 0_- ， 反映了如下事实: 对于无质量粒子，谱函数在零谱值处包含一个 δ 函数，即 $\delta\left(\lambda\right)$ 。

The second identity of (104) follows in the presence of Lorentz symmetry. For example, at finite temperature or density, this step cannot be performed as the heat bath or the medium singles out a rest frame. Here, we consider Lorentzian quantum gravity in an expansion about the flat background with full Lorentz symmetry. Then, the propagator is a function of momentum-squared, p^2 .

式 (104) 的第二个恒等式在洛伦兹对称性成立时得到。例如，在有限温度或有限密度下，这一步无法成立，因为热库或介质会区分出一个静止系。本文我们研究的是在平直背景上展开、具有完整洛伦兹对称性的洛伦兹量子引力。因此，传播子是动量平方 p^2 的函数。

Note that the KL representation (104) does not necessarily hold for unphysical fields such as the graviton or the gluon, but the spectral function is defined in any case by the discontinuity of the propagator on the timelike axis with

请注意，KL 表示 (104) 并不一定适用于引力子或胶子这类非物理场，但在任何情况下，谱函数都由类时轴上传播子的不连续性定义

$$\rho\left(\lambda, |\mathbf{p}|\right) = \lim_{\varepsilon \to 0} 2\, \mathrm{Im}\, G\left(p_0 = -\mathrm{i}\left(\lambda + \mathrm{i}\varepsilon\right), |\mathbf{p}|\right). \tag{105}$$

The spectral function acts as a linear response function of the two-point correlator, encoding the energy spectrum of the theory. For fields that present asymptotic states, the existence of the KL representation can be readily shown with a simple insertion of a complete set of states with masses λ^2 and propagators $1/\left(\lambda^2 + p^2\right)$

into the two-point correlation function of the fields. Then, the spectral function can be understood as a probability density for the transition to an excited state with energy λ . Accordingly, in this case, the spectral function is positive semi-definite $\rho(\lambda) \geq 0$ and the total spectral weight can be normalized to unity:

> 谱函数充当两点关联函数的线性响应函数，编码了理论的能谱。对于存在渐近态的场，只需将质量为 λ^2 的完备态集合与传播子 $1/(\lambda^2+p^2)$ 插入场的两点关联函数，即可轻松证明 KL 表示的存在。此时，谱函数可理解为跃迁到能量为 λ 的激发态的概率密度。因此，在这种情况下，谱函数是半正定的 $\rho(\lambda) \geq 0$ ，总谱权重可归一化为 1:

$$\int_{0_-}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \lambda \rho(\lambda) = 1, \quad \text{with} \quad \rho(\lambda) > 0. \tag{106}$$

Equation (106) holds true if the dressing of the Euclidean propagator is normalized to unity in the UV, $\lim\limits_{p^2\to\infty} p^2 G\left(p^2\right) = 1$, which encodes the canonical commutation relations. The total spectral weight and the analytic IR and UV limits have been discussed in detail in [90, 92, 95] .

> 若欧几里得传播子的修饰在紫外归一化为 1，等式 (106) 成立，$\lim\limits_{p^2\to\infty} p^2 G\left(p^2\right) = 1$ ，它编码了正则对易关系。总谱权重以及解析的红外和紫外极限已在 [90, 92, 95] 中详细讨论。

A schematic example of a spectral function and the corresponding propagator are displayed in Fig. 11. In the left panel, we see the one-particle delta-peak at the spectral value $\omega^2 = m^2$ and the onset of the multiparticle continuum at $\omega^2 = (2m)^2$. Correspondingly, we show the singularity structure of the propagator in the complex momentum plane in the right panel of Fig. 11: the propagator has a singularity at $q^2 = m^2$, which corresponds to the delta-peak in the spectral function, and a branch cut starting at $q^2 = (2m)^2$ corresponding to the multiparticle continuum.

> 谱函数及其对应传播子的示意图如图 11 所示。左 panel 中，我们可以看到单粒子 delta 峰位于谱值 $\omega^2 = m^2$ 处，多粒子连续谱起始于 $\omega^2 = (2m)^2$ 。相应地，图 11 右 panel 给出了传播子在复动量平面的奇点结构: 传播子在 $q^2 = m^2$ 处存在一个奇点，对应谱函数中的 delta 峰，从 $q^2 = (2m)^2$ 开始的分支切割对应多粒子连续谱。

The above considerations also extend to the spectral representation of higher correlation functions. However, they are far more complicated and even their closed form for higher-order correlation functions such as four-point functions is not completely worked out. Still, the approximations used so far in asymptotically safe gravity are well covered with the known spectral representations of correlation functions in the spectral approach.

> 上述考虑也可以推广到高阶关联函数的谱表示。不过，高阶关联函数要复杂得多，即便是四点函数这类高阶关联函数的闭合形式也尚未完全推导得出。尽管如此，渐近安全引力中目前使用的近似都完全适用于谱方法中已知的关联函数谱表示。

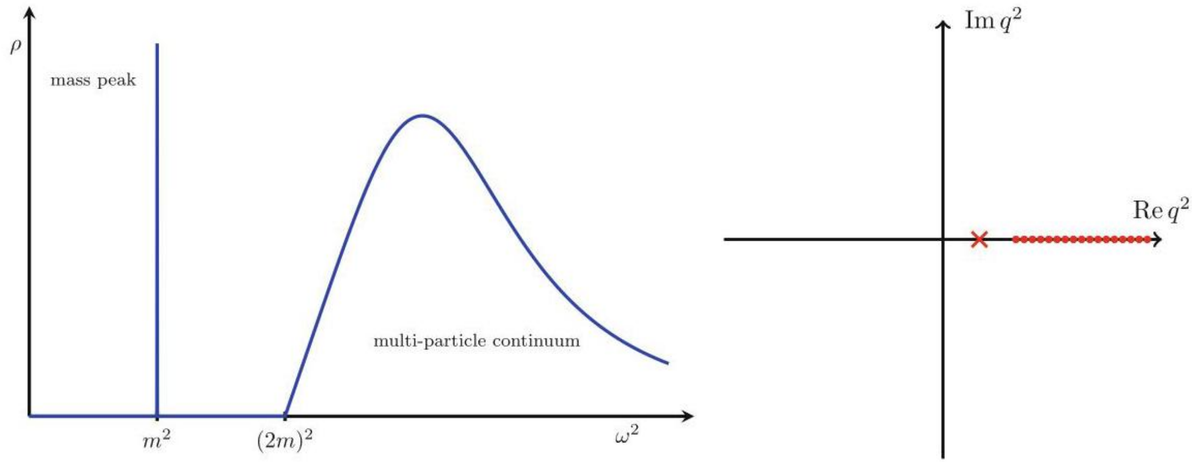


Fig. 11 Schematic illustration of the spectral function (left) and the corresponding propagator in the complex momentum plane (right) for a stable physical particle of mass m . The on-shell mass peak in the spectral function corresponds to a singularity in the propagator (red cross), and the multiparticle continuum corresponds to a branch cut (red line)

图 11 质量为 m 的稳定物理粒子的谱函数 (左) 和复动量平面中对应传播子 (右) 的示意图。谱函数中的在壳质量峰对应传播子的一个奇点 (红叉)，多粒子连续谱对应分支切割 (红线)

The Graviton Spectral Function

引力子谱函数

The explicit computations in the spectral fRG approach have so far been done in an expansion about the flat Minkowski background $\eta_{\mu\nu}$ with (62):

到目前为止，谱函数泛函重整化群 (fRG) 方法的显式计算都是在平直闵氏背景 $\eta_{\mu\nu}$ 下，结合式 (62) 展开完成的:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{Z_h G_{\mathrm{N}}}\, h_{\mu\nu},, \quad \text{with} \quad \eta = \mathrm{diag}\,(1, \mathbf{-1})\,. \tag{107}$$

In the following, we present results for the gauge-fixing condition (9) with $\alpha = \beta = 1$ (harmonic Feynman gauge). The graviton propagator in the flat background has the parametrization:

下文我们给出规范固定条件 (9) 在 $\alpha = \beta = =1$(调和费曼规范) 下的结果。平直背景下的引力子传播子参数化形式为:

$$G_{hh,\mu\nu\rho\sigma}\,(p) = 32\pi G^{(tt)}_{hh}\,(p)\, \Pi^{(tt)}_{\mu\nu\rho\sigma}\,(p) - 64\pi G^{(s)}_{hh}\,(p)\, \Pi^{(s)}_{\mu\nu\rho\sigma}\,(p) + \cdots, \tag{108}$$

where $G^{(tt/s)}_{hh}\,(p)$ are the scalar dressings of the tt-part with five degrees of freedom and the physical scalar part with one degree of freedom, respectively. Further terms are included in the dots and comprise the vector and other scalar modes that are gauge degrees of freedom. In the following, we assume uniform scalar dressing functions which are determined by the dominant tt-mode. This leads to the approximation:

其中 $G_{hh}^{(tt/s)}(p)$ 分别对应含 5 个自由度的 tt 部分标量修饰，以及含 1 个自由度的物理标量部分标量修饰。省略号包含其余项，涵盖作为规范自由度的矢量模和其他标量模。下文我们假设由主导 tt 模确定均匀标量修饰函数，由此得到近似:

$$G_{hh}^{(tt)}(p) = G_{hh}(p)\,,\; G_{hh}^{(s)}(p) = G_{hh}(p)\,, \tag{109}$$

and similarly for the other modes. In the approximation (109), the graviton propagator has a uniform scalar dressing G_{hh} , which can be written in terms of the KL representation (104).

其他模也满足类似关系。在近似 (109) 下，引力子传播子具有均匀标量修饰 G_{hh} ，可以用 KL 表示 (104) 写出。

Before we discuss the full non-perturbative computation and results for the spectral function, we proceed with a discussion of the spectral function for different effective theories of gravity. This will allow us to embed and interpret the full quantum results. We start with the classical Einstein-Hilbert action (2) with a vanishing cosmological constant, $\Lambda = 0$. Then, the flat background is a solution of the equations of motion, and the scalar propagator G_{hh} is simply $1/p^2$ and the corresponding spectral function is a δ -function at vanishing spectral value, to wit,

在讨论谱函数的完整非微扰计算与结果之前，我们先讨论不同有效引力理论的谱函数，这能帮助我们嵌入并解释完整量子结果。我们从宇宙学常数为零的经典爱因斯坦-希尔伯特作用量 (2) 出发，即 $\Lambda = 0$ 。此时平直背景是运动方程的解，标量传播子 G_{hh} 形式即为 $1/p^2$ ，对应谱函数是谱值为零处的 δ 函数，即

$$G_{hh}(p) = \frac{1}{p^2},\; \rho_h(p) = \frac{2\pi}{\lambda}\delta(\lambda)\,. \tag{110}$$

Conversely, starting from the higher-derivative Stelle action (6), we find that after a partial fraction decomposition, the transverse-traceless graviton propagator and the corresponding spectral function are given by

反之，从高阶导数施泰勒作用量 (6) 出发，我们发现经部分分式分解后，无迹横引力子传播子与对应谱函数形式为

$$G_{hh}(p) = \frac{1}{p^2} - \frac{1}{p^2 + M_{\text{ghost}}^2},\; \rho_h(p) = 2\pi\left[\frac{1}{\lambda}\delta(\lambda) - \frac{1}{M_{\text{ghost}}}\delta\left(M_{\text{ghost}} - \lambda\right)\right].$$

(111)

The second delta function in the spectral function in (111) comes with a negative sign indicating that this is a ghost state. The mass of the ghost is given by $M_{\text{ghost}} = M_{\text{pl}}/\sqrt{32\pi b}$, which is of the order of the Planck mass. Depending on the sign of b , the state can also be tachyonic. This is a reflection of the unitarity problems in higher-derivative gravity, commonly known as Ostrogradsky instabilities. Note that only the prefactor of the Weyl-squared tensor enters in the propagator since we are only looking at the transverse-traceless part.

The Ricci scalar-squared term would enter in the physical scalar mode giving rise to a massive scalar mode that can be used for inflation à la Starobinsky [301]; for reviews see, e.g., [302-304] and for applications in asymptotically safe gravity [12,305-309].

谱函数 (111) 中第二个 δ 函数带负号，说明这是一个鬼态。鬼的质量由 $M_{\text{ghost}} = M_{\text{pl}}/\sqrt{32\pi b}$ 给出，量级为普朗克质量。根据 b 符号的不同，该态也可能成为快子。这反映了高阶导数引力中的幺正性问题，即众所周知的奥斯特罗格拉德斯基不稳定性。注意，由于我们仅研究无迹横部分，传播子中仅出现外尔平方张量的 prefactor。里奇标量平方项会作用在物理标量模上，产生一个可用于斯塔罗宾斯基暴胀的大质量标量模 [301]；综述参见例如 [302-304]，渐近安全引力中的应用参见 [12,305-309]。

We can go beyond the classical graviton spectral function by treating gravity as an effective field theory with the Planck scale as cutoff. The one-loop effective action reads

我们可以将引力视为以普朗克能标为截断的有效场论，突破经典引力子谱函数的范围。单圈有效作用量形式为

$$\Gamma_{1\text{-loop}} = \int_x \sqrt{g}\left(\frac{R-2\Lambda}{16\pi G_{\text{N}}} + c_1 R \ln\left(\Box/M_{\text{pl}}^2\right) R + c_2 C_{\mu\upsilon\rho\sigma} \ln\left(\Box/M_{\text{pl}}^2\right) C^{\mu\upsilon\rho\sigma} + \dots\right),$$

(112)

where $\Box = -g^{\mu\upsilon}\nabla_\mu\nabla_\upsilon$ is the d'Alembertian. The prefactors c_1 and c_2 can be determined analytically, and they depend on the two gauge-fixing parameters α and β [92,195]. The coefficient c_2 enters in the transverse-traceless mode, while c_1 enters in the scalar mode. The explicit form of c_2 is, for example, displayed in equation (F3) in [92]. With this one-loop effective action, the inverse graviton propagator takes the form

其中 $\Box = -g^{\mu\upsilon}\nabla_\mu\nabla_\upsilon$ 是达朗贝尔算符。prefactor c_1 和 c_2 可以解析求解，二者依赖两个规范固定参数 α 和 β [92,195]。系数 c_2 作用在无迹横模，c_1 作用在标量模。例如 c_2 的显式形式已展示在文献 [92] 的式 (F3) 中。结合该单圈有效作用量，逆引力子传播子形式为

$$\mathcal{G}_{hh}(p)^{-1} \sim p^2 + \tilde{c}_2 \ln\left(p^2\right) p^4 + \dots, \tag{113}$$

with $\tilde{c}_2 = 32\pi G_{\text{N}} c_2$ and the corresponding spectral function

其中 $\tilde{c}_2 = 32\pi G_{\text{N}} c_2$，对应谱函数为

$$\rho_h(\lambda) \sim \frac{1}{\lambda}\delta(\lambda) + \tilde{c}_2 + \dots \tag{114}$$

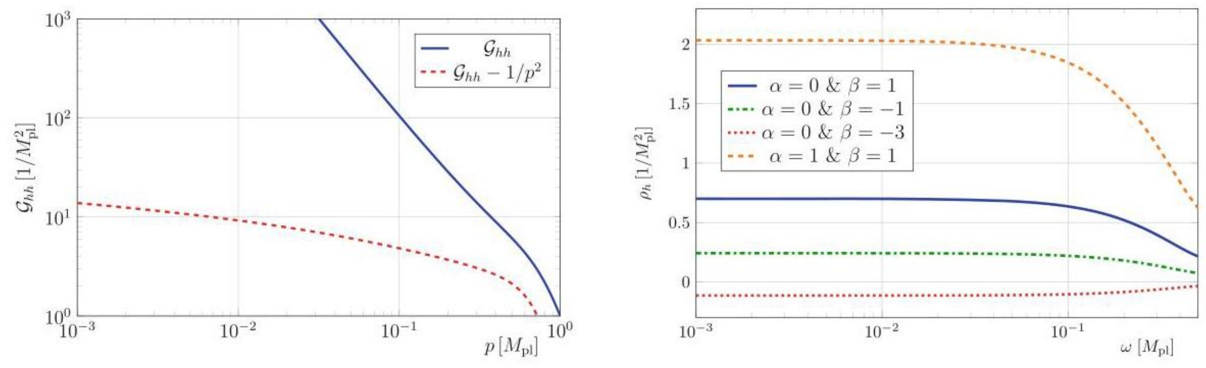


Fig. 12 The graviton propagator (left) and spectral function (right) below the Planck scale computed from one-loop effective field theory. The spectral function contains a gauge-invariant delta peak at vanishing frequencies (not shown in the plot) corresponding to $1/p^2$ behavior in the propagator. The one-loop contribution induces a subleading logarithm in the propagator corresponding to a constant part in the spectral function below the Planck scale

图 12 普朗克能标以下由单圈有效场论计算得到的引力子传播子 (左) 与谱函数 (右)。谱函数在零频处存在规范不变的 δ 峰 (图中未展示)，对应传播子的 $1/p^2$ 行为。单圈贡献在传播子中诱导出次领头对数项，对应普朗克能标以下谱函数中的常数部分。

The graviton propagator and spectral function are displayed in Fig. 12. The left panel shows the propagator. The dominant contribution is the standard $1/p^2$, and the first subleading contribution is the logarithmic $\ln p^2$ contribution. The latter leads to a constant part in the spectral function, displayed in the right panel of Fig. 12. The magnitude of the constant part is directly given by the coefficient c_2 from the one-loop effective action (112). The coefficient c_2 is gauge-dependent, as explained before. We display the result for commonly used values for the gauge-fixing parameters. We intentionally also include the values $\alpha = 0$ and $\beta = -3$ where the coefficient c_2 is negative, and consequently the spectral function has a negative part. This emphasizes that even an un-truncated result of the graviton spectral function is gauge-dependent, and for some values of the gauge-fixing parameter, the result will contain negative parts. The one-loop EFT graviton propagator contains additional cuts and poles in complex momentum plane beyond the Planck scale. These cuts and poles are outside of the validity regime of the effective field theory.

引力子传播子与谱函数如图 12 所示。左图为传播子，其主导贡献是标准的 $1/p^2$ ，第一阶次主导贡献是对数 $\ln p^2$ 项。后者在谱函数中产生常数部分，如图 12 右图所示。常数部分的大小直接由单圈有效作用量 (112) 中的系数 c_2 给出。如前文所述，系数 c_2 依赖于规范，我们给出了常用规范固定参数取值对应的结果。我们特意加入了系数 c_2 为负的取值 $\alpha = 0$ 和 $\beta = -3$ ，在这种情况下谱函数存在负区间。这表明，即使是未截断的引力子谱函数结果也依赖于规范，对于部分规范固定参数取值，结果会包含负区间。单圈有效场论引力子传播子在普朗克能标以外的复动量平面存在额外割线和极点，这些割线和极点都在有效场论的适用范围之外。

We turn now to the computation of the full non-perturbative spectral function from [82] based on the spectral renormalization group (36) with the Callan-Symanzik regulator reviewed in section "Lorentzian Flow Equation for the Effective Action." For the graviton propagator, the scalar part of the CS regulator, (35), reads

我们现在基于“有效作用量的洛伦兹流方程”一节回顾的带 Callan-Symanzik 正则化的谱重整化群 (36)，计算文献 [82] 中给出的完整非微扰谱函数。对于引力子传播子，CS 正则化的标量部分 (35) 形式为

$$R_h = Z_h k^2, \tag{115}$$

where the inverse wave function $1/Z_h$ is the amplitude of the pole contribution of the graviton spectral function:

其中逆波函数 $1/Z_h$ 是引力子谱函数极点贡献的振幅:

$$\rho_h(\lambda) = \frac{1}{Z_h}\left[2\pi\delta\left(\lambda^2 - m_h^2\right) + \theta\left(\lambda^2 - 4m_h^2\right) f_h(\lambda)\right], \tag{116}$$

with $Z_h \equiv Z_h\left(p^2 = -m_h^2\right), m_h^2 = k^2(1+\mu)$ is the graviton mass parameter, and f_h is the multi-graviton continuum.

式中 $Z_h \equiv Z_h\left(p^2 = -m_h^2\right), m_h^2 = k^2(1+\mu)$ 为引力子质量参数，f_h 为多引力子连续谱。

In summary, we are led to the flow of the graviton spectral function:

综上，我们得到引力子谱函数的流方程:

$$\partial_t \rho_h = -2\,\mathrm{Im}\, G_{hh}^2 \left(\partial_t \Gamma_{\mathrm{TT}}^{(hh)} + \partial_t R_h\right), \tag{117}$$

and the respective one of the ghosts. These flows are supplemented with a flow equation for the Newton coupling from the graviton three-point function at vanishing momentum. This leads to the Lorentzian asymptotically safe UV fixed point:

以及鬼场对应的流方程。这些流方程补充了零动量处引力子三点函数给出的牛顿耦合流方程，由此得到洛伦兹渐近安全紫外不动点:

$$(g, \eta_h, \mu)\big|_* = (1.06, 0.96, -0.34)\,. \tag{118}$$

The scaling exponents $\theta = 2.49 \pm 3.17\mathrm{i}$ compare well with those found in Euclidean studies. From the UV fixed point (118), a trajectory was found that connects to standard GR behavior in the IR. The counterterm action in (36) reabsorbs divergences of the flow. We have to introduce a renormalization condition for each divergence, just as in perturbation theory, and make sure that there are only finitely many renormalization conditions. This analysis was done in [83], and it was shown that if the Newton couplings become asymptotically safe, corresponding to a $G_{\mathrm{N}}(p) \to 1/p^2$ in the UV, then there are finitely many renormalization conditions and the theory is predictive. In the absence of a UV fixed point, this analysis would fall back onto the standard non-predictivity found in perturbation theory.

标度指数 $\theta = 2.49 \pm 3.17\mathrm{i}$ 与欧几里得研究得到的结果吻合良好。从紫外不动点 (118) 出发，我们找到了一条能在红外区连通到广义相对论标准行为的轨迹。(36) 中的 counter-term 作用量重新吸收了流的发散。和微扰论一样，我们需要对每个发散引入一个重整化条件，并保证重整化条件的数量有限。该分析已在文献 [83] 中完成，结果表明若牛顿耦合达到渐近安全，对应紫外区的 $G_{\mathrm{N}}(p) \to 1/p^2$ ，则重整化条件数量有限，理论具有预言性。如果不存在紫外不动点，该分析就会退化为微扰论中典型的无预言性结果。

On this trajectory, the multi-graviton continuum can be integrated. The result of the computation is displayed in Fig. 13. The graviton spectral function, displayed in the left panel of Fig. 13, has a delta-peak at vanishing frequencies corresponding to the massless graviton and a positive ensuing multi-graviton continuum. In the IR, the spectral function is constant as expected from the EFT considerations, and in the UV, the spectral function decays with an asymptotically safe power law $\lambda^{\eta_h^*-2}$. The corresponding Euclidean propagator is shown in the right panel of Fig. 13. In the IR, the leading order behavior is the standard GR $1/p^2$ behavior, with a subleading logarithmic $\ln\left(p^2\right)$ behavior from the EFT corrections. In the UV, we encounter the same asymptotically safe scaling $p^{\eta_h^*-2}$. The constant part of the spectral function in the IR is universal (regulator-independent) but gauge-dependent [92, 310] . It can be determined within effective theory, giving $61/30 \approx 2.03$. In [82], the value $70\pi/\left(9\sqrt{3}\right) - 11 \approx 3.11$ was found, since the feedback of f_h onto the flow was neglected. In comparison, in the analytic continuation in [92], the exact EFT value was found: 7/10 in the gauge $\alpha = 0$ and $\beta = 1$.

在该轨迹上，可以对多重引力子连续谱进行积分。计算结果如图 13 所示。图 13 左图的引力子谱函数在零频率处存在一个对应无质量引力子的 δ 峰，其后是正的多重引力子连续谱。在红外区域，谱函数如有效场论预期保持恒定；在紫外区域，谱函数以渐近安全幂律 $\lambda^{\eta_h^*-2}$ 衰减。图 13 右图给出了对应的欧几里得传播子。在红外区域，领头阶行为是标准广义相对论 $1/p^2$ 行为，有效场论修正带来次领头对数 $\ln\left(p^2\right)$ 行为。在紫外区域，我们得到了相同的渐近安全标度 $p^{\eta_h^*-2}$ 。谱函数在红外区域的常数部分是普适的 (与正则化无关)，但依赖于规范 [92, 310] 。它可以在有效场论框架下确定，结果为 $61/30 \approx 2.03$ 。文献 [82] 中，由于忽略了 f_h 对流的反馈，得到的结果为 $70\pi/\left(9\sqrt{3}\right) - 11 \approx 3.11$ 。相比之下，文献 [92] 的解析延拓得到了精确的有效场论值: 规范 $\alpha = 0$ 下为 7/10 ，规范 $\beta = 1$ 下为 $\beta = 1$ 。

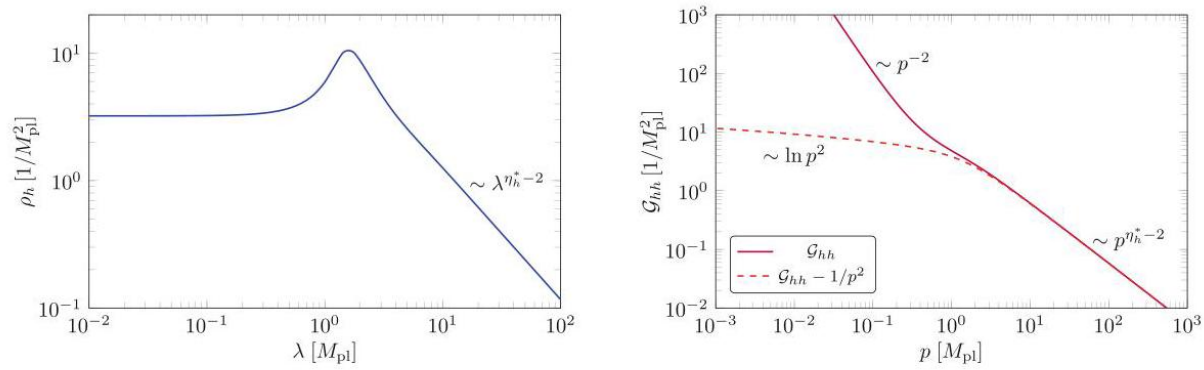


Fig. 13 The full graviton spectral function (left) and propagator (right). Below the Planck scale, they match the effective field theory, while they display an asymptotically safe scaling above the Planck scale. (Taken from [82])

图 13 完整引力子谱函数 (左) 与传播子 (右)。普朗克能标以下，二者与有效场论匹配；普朗克能标以上，二者展现渐近安全标度。(引自 [82])

We know from the discussion of the EFT results (see Fig. 12) that the graviton propagator and the graviton spectral function are gauge-dependent. Figure 13 shows the result in the gauge $\alpha = \beta = 1$. Despite the gauge dependence, the result is very encouraging: the existence of a KL spectral representation in any gauge is already nontrivial. Furthermore, the propagator does not have any tachyonic and ghostlike instabilities and is therefore compatible with all requirements from unitarity; see also [311] for a discussion. The latter is expected to be a gauge-independent feature.

我们从有效场论结果的讨论 (见图 12) 可知，引力子传播子和引力子谱函数都依赖于规范。图 13 给出了规范 $\alpha = \beta = 1$ 下的结果。尽管存在规范依赖性，结果仍十分令人振奋:KL 谱表示在任意规范下都成立，这已经是非平凡的结论。此外，传播子不存在快子和鬼类不稳定性，因此满足幺正性的所有要求，相关讨论参见文献 [311]。后者预期是一个与规范无关的性质。

Finally, the spectral function shown in Fig. 13 can be inserted in the spectral representation of the graviton, (104), giving access to the propagator in the whole complex momentum plane. The real and imaginary parts of the propagator are depicted in Fig. 14, where we excluded the pole contribution in the real part. Both parts vanish for asymptotically large p . The real part displays a unique pole at vanishing p not shown in Fig. 14, while the imaginary part shows a branch cut along the timelike axis.

最后，可将图 13 中的谱函数代入引力子的谱表示 (104)，即可得到整个复动量平面上的传播子。传播子的实部和虚部如图 14 所示，其中我们在实部中去掉了极点贡献。两部分在渐近大 p 下都趋于零。实部在零 p 处存在唯一极点 (未在图 14 中展示)，虚部沿类时轴存在分支切割。

Toward Curved Backgrounds

朝向弯曲背景

We close this section with a discussion of Lorentzian quantum gravity with a nonvanishing cosmological constant, $\Lambda \neq 0$. This amounts to the IR values:

我们在本节最后讨论带有非零宇宙学常数的洛伦兹量子引力，$\Lambda \neq 0$ 。这对应于红外取值:

$$\left(G_{\mathrm{N}}(k), Z_h(k), k^2\mu(k)\right)\Big|_{k\to 0} = \left(G_{\mathrm{N}}, 1, -2\Lambda\right), \tag{119}$$

for the dimensionful Newton constant and cosmological constant, respectively. The value of the wave function or residue value, Z_h , is normalized to unity.

分别对应量纲化牛顿常数和宇宙学常数。波函数取值即留数取值 Z_h 归一化为 1。

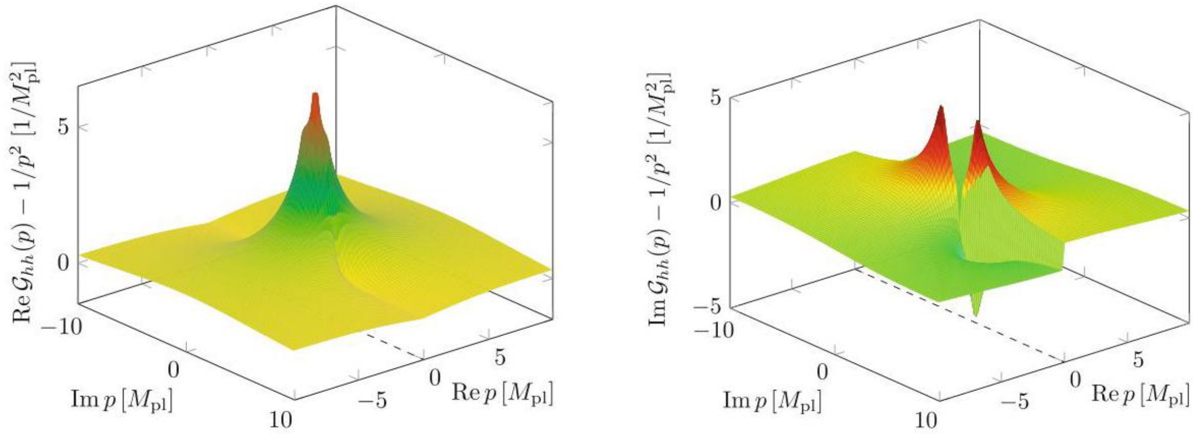


Fig. 14 Real and imaginary parts of the graviton propagator in the complex plane. The dashed line indicates the timelike axis. (Taken from [82])

图 14 复平面中引力子传播子的实部与虚部。虚线表示类时轴。(引自文献 [82])

The following results have been obtained within the expansion about the flat Minkowski background used for vanishing cosmological constant, $\Lambda = 0$, for which the flat background is a solution to the Einstein equations. For a nonvanishing cosmological constant, the flat background is not a solution to the Einstein equations and should be understood as a background field such as in quantum electrodynamics (QED) with a background electric or magnetic field. It is well-known that QED in such a background does not represent a closed system and hence unitarity and probability conservation is potentially lost.

以下结果是在零宇宙学常数所用的平坦闵氏背景展开框架下得到的，$\Lambda = 0$ ，该框架下平坦背景是爱因斯坦方程的解。对于非零宇宙学常数，平坦背景不再是爱因斯坦方程的解，应当将其理解为类似于量子电动力学 (QED) 中存在背景电场或磁场时的背景场。众所周知，这种背景下的 QED 不是封闭系统，因此幺正性和概率守恒可能不成立。

We continue our discussion, bearing the above in mind. On de Sitter (dS) or anti-de Sitter (AdS) backgrounds, the classical graviton and ghost are still massless, while the graviton vertices are deformed in comparison with flat backgrounds. We expect to find modifications of the spectral function at small spectral values since alterations of the geometry are relevant for large spatial distances and are negligible at small distances. As discussed above, we still expand about the flat Minkowski background, and hence our setup is an off-shell expansion at $\Lambda \neq 0$. For simplified trajectories

我们牢记上述结论继续讨论。在德西特 (dS) 或反德西特 (AdS) 背景下，经典引力子和鬼仍然是无质量的，但与平坦背景相比引力子顶点发生了形变。我们预计谱函数在低谱值处会发生修正，因为几何改变对大空间距离有显著影响，在小距离可忽略。如前文所述，我们仍然围绕平坦闵氏背景展开，因此我们的框架是 $\Lambda \neq 0$ 处的离壳展开。对于简化轨迹

$$G_{\mathrm{N}}(k) = \frac{g^*}{k^2 + g^* M_{\mathrm{pl}}^2}, \tag{120}$$

the spectral flows admit analytic solutions. We also neglect the ghost contributions and in combination this facilitates the present qualitative discussion.

谱流存在解析解。我们同时忽略了鬼贡献，二者结合方便了本文的定性讨论。

The UV fixed point value g^* of the Newton constant in (120) is a free parameter, and the respective UV fixed point is given by

(120) 式中牛顿常数的紫外不动点取值 g^* 是一个自由参数，对应的紫外不动点由下式给出

$$\mu^* = \frac{-g^*}{c_\mu + g^*}, \; \eta_h^* = \frac{2g^*}{2c_\eta + g^*}, \tag{121}$$

with $(c_\mu, c_\eta) = (1.77, 0.49)$ known analytically; for more details, see [82]. Using $g^* = 1.06$ from (118), we find $\mu^* = -0.38$ and $\eta_h^* = 1.04$. Both values are approximately 10% off (see (118)) which supports the present qualitative approximation. In particular, it indicates that the ghost contributions are indeed subleading. Moreover, the anomalous dimension of the fluctuating graviton is positive and takes values in the range $\eta_h^* \in (0, 2)$. This allows for a positive spectral function for the fluctuating graviton in the presence of a nonvanishing cosmological constant as for $\Lambda = 0$.

其中 $(c_\mu, c_\eta) = (1.77, 0.49)$ 有解析形式；更多细节参见文献 [82]。利用 (118) 式中的 $g^* = 1.06$ ，我们得到 $\mu^* = -0.38$ 和 $\eta_h^* = 1.04$ 。两个取值与精确结果相差约 10%(见 (118) 式)，这支持了本文的定性近似。尤其这说明鬼贡献确实是次领头阶的。此外，涨落引力子的反常维数为正，取值范围为 $\eta_h^* \in (0, 2)$ 。这使得在存在非零宇宙学常数时，涨落引力子的谱函数为正，正如 $\Lambda = 0$ 的情况。

The flow is readily integrated analytically with the IR boundary conditions (119):

利用红外边界条件 (119)，可以很方便地对流做解析积分:

$$\begin{aligned} Z_h(k) &= \left(1 + \frac{1}{c_\eta \eta_h^*} \frac{k^2}{M_{\text{pl}}^2}\right)^{-\frac{1}{2}\eta_h^*}, \\ \mu(k) &= \mu^* - \frac{2\Lambda}{k^2} + \frac{c_1 M_{\text{pl}}^2 - 2\Lambda}{k^2} \left[Z_h(k)^{-c_2} - 1\right], \end{aligned} \tag{122}$$

with $c_1 = 2.17g^*/(1.77 + g^*)$ and $c_2 = 0.45$; again we refer to [82] for the details. The UV tail of the solution (122) has the same qualitative behavior as the full solution for $\Lambda = 0$ displayed in Fig. 13, and the differences can be traced back to the additional approximations used here. Remarkably, the short-distance mass parameter is constrained within the narrow range $\mu^* \in (-1, 0)$ and only takes negative values. Evidently, the mass parameter $\mu(k)$ interpolates smoothly between μ^* in the UV and the cosmological constant $-2\Lambda/k^2$ in the IR. In summary,(120) and (122) are viable approximate solutions interpolating between an asymptotically safe fixed point and general relativity with a cosmological constant.

其中包含 $c_1 = 2.17g^*/(1.77 + g^*)$ 和 $c_2 = 0.45$ ；细节请再次参见文献 [82]。解 (122) 的紫外尾部与图 13 中展示的 $\Lambda = 0$ 的完整解具有相同的定性行为，差异可以追溯到本文使用的额外近似。值得注意的是，短距离质量参数被约束在狭窄范围 $\mu^* \in (-1, 0)$ 内，且仅取负值。显然，质量参数 $\mu(k)$ 在紫外的 μ^* 和红外的宇宙学常数 $-2\Lambda/k^2$ 之间平滑插值。综上，(120) 和 (122) 是可行的近似解，能够在渐近安全不动点和带宇宙学常数的广义相对论之间平滑插值。

The results for the spectral function in a de Sitter or anti-de Sitter background with $\Lambda \neq 0$ are depicted in Fig. 15. We find that a cosmological constant $\Lambda \neq 0$ does not affect the spectral function for spectral values above $\lambda \gtrsim \sqrt{|8\Lambda|}$. This is in line with our expectation that the geometry should not leave an imprint for larger spectral values. In turn, for smaller spectral values, the geometry leaves such an imprint. For AdS backgrounds, the cosmological constant acts like a mass term which leads to a suppression. Conversely, the spectral function is enhanced for dS backgrounds because $\Lambda > 0$ acts like a negative mass-squared term.

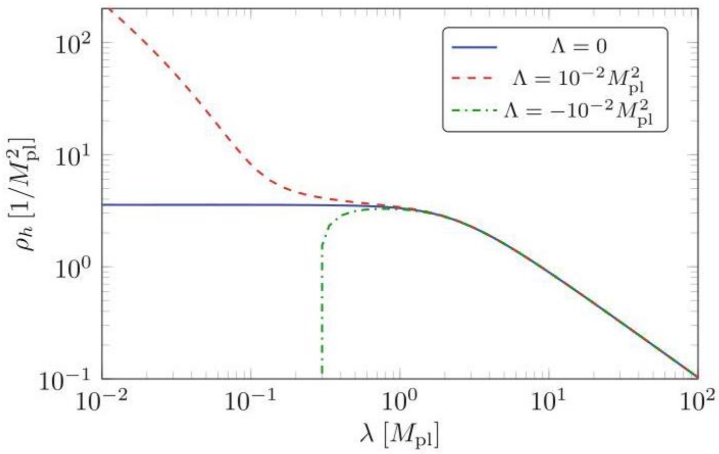
我们在图 15 中描绘了德西特背景或反德西特背景下，含 $\Lambda \neq 0$ 的谱函数结果。我们发现，宇宙学常数 $\Lambda \neq 0$ 不会影响谱值大于 $\lambda \gtrsim \sqrt{|8\Lambda|}$ 区域的谱函数，这符合我们的预期: 更大谱值区域不会留下几何印记。反之，较小谱值区域会留下几何印记。对于反德西特 (AdS) 背景，宇宙学常数的作用类似于质量项，会产生抑制效应。相反，德西特 (dS) 背景的谱函数会增强，因为 $\Lambda > 0$ 的作用类似于负质量平方项。

The off-shell effects due to the cosmological constant become even more pronounced if the ghost contributions are retained. The ghost remains on-shell at k^2 compared to the off-shell graviton at $m_h^2 = k^2(1+\mu)$. We find that for AdS backgrounds (at $\mu = 3$), off-shell gravitons can directly scatter into the on-shell multi-ghost continuum and the flow of f_h diverges, while it stays finite for dS backgrounds. As already mentioned before, in this off-shell computation, the flat Minkowski background bears similarities to an external electric or magnetic field in QED. External backgrounds or boundary conditions can introduce driving forces or friction that constantly feed or suppress scattering processes, which then destroy unitarity much like in open quantum systems. This analogy allows for a heuristic interpretation of the AdS singularity in the flow: there the off-shell background serves as a driving force for graviton scattering processes. We expect that full on-shell AdS flows with ghost contributions remain finite. Then, graviton and ghost are both on-shell massless, and it is the off-shell shift of mass scales that triggers the divergence.

如果保留鬼场贡献，宇宙学常数引起的离壳效应会更加显著。与处于离壳的 $m_h^2 = k^2(1+\mu)$ 引力子相比，鬼场始终保持在 k^2 的在壳状态。我们发现，对于 AdS 背景 (在 $\mu = 3$ 处)，离壳引力子可以直接散射到在壳多鬼连续谱，导致 f_h 的流发散，而 dS 背景下的流始终有限。如前所述，在该离壳计算中，平直闵氏背景与量子电动力学中的外电场或外磁场具有相似性。外背景或边界条件可以引入持续驱动或散射过程的驱动力或阻尼，进而类似开放量子系统破坏幺正性。这一类比可以对流中的 AdS 奇点给出启发式解释:AdS 中的离壳背景是引力子散射过程的驱动力。我们预计，包含鬼场贡献的完整在壳 AdS 流依然保持有限，此时引力子和鬼场都是在壳无质量的，触发发散的是质量标度的离壳偏移。

Fig. 15 Enhancement (or suppression) of the spectral function due to a positive (or negative) cosmological constant. (Taken from [82])

图 15 正 (负) 宇宙学常数对谱函数的增强 (或抑制) 作用。(引自文献 [82])



In summary, we consider the result in the presence of a nonvanishing cosmological constant as very promising. However, in our opinion, a fully conclusive analysis requires the expansion about a solution of the (quantum) Einstein equation and hence the expansion about a nontrivial background such as done in the Euclidean setting in [88].

总而言之，我们认为存在非零宇宙学常数时得到的结果很有前景。但在我们看来，要得到完全确凿的分析，需要围绕 (量子) 爱因斯坦方程的解展开，也就是围绕非平凡背景展开，就像文献 [88] 在欧氏框架下所做的那样。

Outlook

展望

We have reviewed the first direct approach to asymptotically safe gravity in space-times with Lorentzian signatures. This is based on the novel spectral renormalization group which allows for non-perturbative computations on space-times with Lorentzian signature. This approach has been used to compute the graviton spectral function, which matches expectations from effective field theory at small momenta and from asymptotic safety at large momenta. The propagator has no ghost or tachyonic instabilities and thus there are no indications of unitarity violation, which is an encouraging first result. Next steps concern the computation of spectral representations of vertices and the direct computation of scattering amplitudes in asymptotically safe quantum gravity.

我们回顾了洛伦兹号差时空中渐近安全引力的第一种直接研究方法。该方法基于全新的谱重整化群，可用于洛伦兹号差时空中的非微扰计算。利用该方法我们计算得到了引力子谱函数，其结果与小动量下有效场理论的预期、大动量下渐近安全的预期均一致。该传播子不存在鬼场或快子不稳定性，因此没有证据表明其存在幺正性破坏，这是一个令人鼓舞的初步结果。下一步研究将围绕顶点谱表示的计算，以及渐近安全量子引力中散射振幅的直接计算展开。

Summary

摘要

In this contribution, we have reviewed the state of the art of the fluctuation approach to quantum gravity. This approach is based on the path integral of metric quantum gravity, which we introduced in section ”Quantum Field Theory Approach to Quantum Gravity.” The key element is to disentangle the correlation functions of the dynamical graviton fluctuation field $h_{\mu\nu}$ with that of the background field $\bar{g}_{\mu\nu}$ within a systematic vertex expansion. For these correlation functions, we set up flow equations on space-times with Euclidean and Lorentzian signatures, and it is the running fluctuation correlation functions that build the core of the renormalization group flow; see section ”Functional Renormalization Group Approach to Metric Quantum Gravity.” The distinction of fluctuation and background field is strictly necessary to fulfill the diffeomorphism symmetry identities, as discussed in section ”Background Independence and Symmetry Identities.” The technical foundations including the underlying concept for the convergence of the expansion scheme are presented in section ”Fluctuation Approach.”

在本文中，我们综述了量子引力涨落方法的研究进展。该方法基于度规量子引力的路径积分，我们已在“量子引力的量子场论方法”一节中对此进行介绍。该方法的核心是在系统性顶点展开中，分离动力学引力涨落场 $h_{\mu\nu}$ 与背景场 $\bar{g}_{\mu\nu}$ 的关联函数。我们为欧几里得号差和洛伦兹号差时空下的这些关联函数建立了流方程，跑动涨落关联函数构成了重整化群流的核心，参见“度规量子引力的泛函重整化群方法”一节。如“背景无关性与对称恒等式”一节所述，严格区分涨落场与背景场是满足微分同胚对称恒等式的必要条件。包含展开方案收敛性基本概念在内的技术基础，已在“涨落方法”一节中给出。

We have highlighted some remarkable recent results in this approach, for example, for the computation of momentum-dependent correlation functions at vanishing cutoff scales, see section ”Momentum-Dependent Correlation Functions”; for the computation of full SM flows including gravity, see section ”The Asymptotically Safe Standard Model”; and for the first direct Lorentzian computation leading to the graviton spectral function, see section ”Lorentzian Quantum Gravity.”

我们重点介绍了该方向一些值得关注的最新成果, 例如: 零截断标度下动量依赖关联函数的计算, 参见“动量依赖关联函数”一节；包含引力的完整标准模型流的计算，参见“渐近安全标准模型”一节；以及首次得到引力子谱函数的直接洛伦兹计算，参见“洛伦兹量子引力”一节。

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